The Fourth International Symposium on

Banach and Function Spaces 2012

September 12–15, 2012
Kyushu Institute of Technology, Tobata Campus
Kitakyushu, Japan

Supported by

Kyushu Institute of Technology
Mathematical Society of Japan
West Japan Industry and Trade Convention Association

Organizers

Tomonari Suzuki (Kyushu Institute of Technology) – Chair
Mikio Kato (Shinshu University) – Vice Chair
Lech Maligranda (Luleå University of Technology, Sweden) – Vice Chair
Yoshiaki Okazaki (Kyushu Institute of Technology) – Vice Chair
Fernando Cobos (Complutense University of Madrid, Spain)
Eiichi Nakai (Ibaraki University)
Kichi-Suke Saito (Niigata University)
Wataru Takahashi (Tokyo Institute of Technology)
Yasuji Takahashi (Okayama Prefectural University)
Kenjiro Yanagi (Yamaguchi University)

Local Organizers

from Kyushu Institute of Technology

Toshiharu Fujita, Michihiro Hirayama, Aoi Honda,
Toshiharu Ikeda, Hiroshi Sakai, Takasi Senba, Tohru Wakasa

September 12 (Wednesday)

0830–	Registration
0915–0925	Opening
0930–1010	Jurgen Appell Bounded variation and around
1010–1050	Fernando Cobos Entropy and extrapolation of compact operators
1110–1150	Wataru Takahashi Existence theorems, duality theorems and convergence theorems for nonlinear operators in Banach spaces
1150–1230	Yoshiaki Okazaki A new sequence space $\Lambda_p(f)$
1400–1430	Hans-Jürgen Schmeisser Moduli of smoothness related to the Laplacian and approximation by Bochner-Riesz families
1430–1500	Shuichi Sato Cesàro and Riesz means of critical order on certain function spaces
1500–1530	Sin-Ei Takahasi Segal algebras which are neither BSE nor BED
1550–1610	Yoshihiro Mizuta Sobolev's inequality for Riesz potentials of functions in grand Morrey spaces of variable exponent
1610–1630	Toshiharu Kawasaki and Masashi Toyoda* On the existence of solutions of second order ordinary differential equations
1630–1650	Oinarov Ryskul Boundedness of integral operators in weighted Sobolev space
1700–1720	Yasunori Kimura* and Satit Saejung Strong convergence of an iterative scheme for two different types of operators
1720–1740	Poom Kumam* and Chirasak Mongkolkeha Some generalized asymptotic pointwise ρ-contraction mappings involving orbits in Modular spaces
1740–1800	Fumiaki Kohsaka Fixed point theorems for some nonlinear mappings related to resolvents of monotone or accretive operators
1900–2100	Welcome Party (Kokura Recent Hotel)

September 13 (Thursday)

0930–1010	Andrew Tonge Matrices and tensors of smallest possible norm
1010–1050	Pier Luigi Papini Diametrically maximal sets in Banach spaces
1110–1150	Anthony To-Ming Lau Fixed point set of measures and positive definite functions on a locally compact group
1150–1220	Witold Wnuk Schur type properties in Banach lattices–a survey
1220-1230	Photo Shoot
1400–1430	Kichi-Suke Saito How to caluculate the James constants of Banach spaces?
1430–1500	Satit Saejung* and Ji Gao Banach spaces which are semi-uniform Kadec–Klee
1500–1530	Mikio Kato Partially ℓ_1-norms and convex functions
1550–1610	Ken-Ichi Mitani Skewness and some geometrical constants of Banach spaces
1610–1630	Ryotaro Tanaka* and Kichi-Suke Saito A structure of n-dimensional normed linear spaces
1630–1650	Takashi Honda The orthogonal decomposition of Banach spaces and non-linear analytic approach to the splitting theorem of Jacobs-deLeeuw-Glicksberg
1700–1720	Katsuo Matsuoka B_σ-function space estimates for some operators
1720–1740	Hiroyasu Mizuguchi*, Kichi-Suke Saito and Ryotaro Tanaka The Dunkl-Williams constant of some Banach spaces
1740–1800	Sachiko Atsushiba Convergence of iterative sequences for nonlinear mappings
1800–1820	Kamthorn Chailuek* and Marisa Senmoh The duality of a generalized Bergman space

September 14 (Friday)

0930–1010	Lech Maligranda Structure of Cesàro function spaces
1010–1050	Guillermo P. Curbera The Cesàro operator on Hardy spaces: extensions and optimal domains
1110–1150	Kazimierz Goebel Some exotic constructions in Banach spaces
1150–1230	Hiro-o Kita Some inclusion relations of Orlicz-Morrey spaces and the Hardy-Littlewood maximal function
1400–1430	Luz M. Fernández-Cabrera Interpolation using the unit square and limiting real methods
1430–1500	Yoshihiro Sawano Hardy spaces with variable exponents
1500–1530	Eiichi Nakai Generalized Morrey spaces and generalized fractional integrals
1550–1610	Kenjiro Yanagi Generalized metric adjusted skew information and uncertainty relation
1610–1630	Shuji Watanabe Applications of fixed point theorems to the BCS gap equation for superconductivity
1630–1650	Hiromichi Ohno Entanglement of marginal tracial states
1700–1720	Koji Aoyama Existence of fixed points of firmly nonexpansive-like mappings in Banach spaces
1720–1740	Takanori Ibaraki Shrinking projection methods for common fixed point problems in a Banach space
1740–1800	Rieko Kubota and Yukio Takeuchi* On Ishikawa's strong convergence theorem
1830–2100	Banquet at Meisen Kaikan Restaurant

September 15 (Saturday)

0930–1010	Jun Kawabe Metrizability of the Lévy topology on nonadditive measures
1010–1040	Hidefumi Kawasaki Discrete fixed point theorems
1040–1110	Yoshikazu Kobayashi*, Naoki Tanaka and Yukino Tomizawa Nonautonomous differential equations in Banach spaces with an application
1130–1150	Tomonari Suzuki Some examples on p-uniform convexity and q-uniform smoothness
1150–1210	Yohei Tsutsui Convolution operators and div-curl lemma on weighted Hardy sapces with an application to Navier-Stokes equations
1210–1230	Yoshifumi Ito New notions of differential calculus of L^p-functions and L^p_{loc}-functions
1400–2000	Short Excursion (Kokura Castle, Castle Garden, Mojiko Harbor)

Sep–12 0930–1010

Bounded variation and around

Jurgen Appell

Department of Mathematics, University of Wurzburg,
Emil-Fischer Str. 30, D-97074 Wurzburg, Germany
Email **jurgen @dmuw.de**

Since Camille Jordan introduced functions of bounded variation in 1881, they have been generalized and applied to many different fields of Real Analysis, Fourier Analysis, Functional Analysis, Geometric Measure Theory, Nonlinear Analysis, and even Mathematical Physics. The aim of the talk is to recall some of those generalizations and to discuss their relation to other function spaces, with a particular emphasis on examples and counterexamples. We also sketch some open problems to provide a glimpse at various directions in which current research is moving. The presentation will be elementary and does not require a profound background in functional analysis or function spaces.

[1] J. Appell, J. Banaś, N. Merentes, Bounded Variation and Around, de-Gruyter, Berlin (to appear 2013).

Entropy and extrapolation of compact operators

Fernando Cobos

Departamento de Análisis Matemático,
Facultad de Ciencias, Universidad Complutense de Madrid,
Plaza de Ciencias 3, 28040 Madrid. Spain.
Email **cobos @ mat.ucm.es**

Let Ω be a bounded domain in $\mathbb{R}^d$ with smooth boundary and with finite Lebesgue measure. Consider the fractional Sobolev space $H^s_p(\Omega)$ on Ω with $0 < s < d$ and $1 < p < d/s$. Let p^* be the critical exponent $1/p^* = 1/p - s/d$. It is known that the natural embedding $H^s_p(\Omega) \hookrightarrow L_q(\Omega)$ is compact for $p < q < p^*$ and that $H^s_p(\Omega) \hookrightarrow L_{p^*}(\Omega)$ is bounded but not compact (see [3]). Several authors have study the problem of finding spaces Y close to $L_{p^*}(\Omega)$ such that the embedding $H^s_p(\Omega) \hookrightarrow Y$ is still compact. We will consider this problem for abstract scales of Banach spaces. Moreover, we will derive quantitative versions of the results in terms of entropy numbers. Applications to limiting Sobolev embeddings are also given.

Results are part of joint papers with Thomas Kühn [1, 2].

[1] F. Cobos and T. Kühn, Extrapolation results of Lions-Peetre type, preprint (2012).

[2] F. Cobos and T. Kühn, Extrapolation estimates for entropy numbers, preprint (2012).

[3] D.E. Edmunds and H. Triebel, "Function Spaces, Entropy Numbers and Differential Operators", Cambridge Univ. Press, Cambridge 1996.

Sep–12 1110–1150

Existence theorems, duality theorems and convergence theorems for nonlinear operators in Banach spaces

Wataru Takahashi

Department of Mathematical and Computing Sciences,
Tokyo Institute of Technology, Meguro-ku, Tokyo 152-8552, Japan
Email **wataru @ is.titech.ac.jp**

Let H be a real Hilbert space. Let C be a nonempty subset of H and let T be a mapping of C into H. We denote by $F(T)$ the set of *fixed points* of T and by $A(T)$ the set of *attractive points* [17] of T, i.e.,

(i) $F(T) = \{z \in C : Tz = z\}$;

(ii) $A(T) = \{z \in H : \|Tx - z\| \leq \|x - z\|, \quad \forall x \in C\}$.

A mapping $T : C \to H$ is said to be *nonexpansive* if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. A mapping $T : C \to H$ is called *nonspreading* [7, 8] if $2\|Tx - Ty\|^2 \leq \|Tx - y\|^2 + \|Ty - x\|^2$ for all $x, y \in C$. A mapping $T : C \to H$ is called *hybrid* [16] if $3\|Tx - Ty\|^2 \leq \|x - y\|^2 + \|Tx - y\|^2 + \|Ty - x\|^2$ for all $x, y \in C$. The class of nonspreading mappings was first defined in a strictly convex, smooth and reflexive Banach space. The resolvents of a maximal monotone operator are nonspreading mappings; see [8] for more details. These three classes of nonlinear mappings are important in the study of the geometry of infinite dimensional spaces. Indeed, by using the fact that the resolvents of a maximal monotone operator are nonspreading mappings, Takahashi, Yao and Kohsaka [20] solved an open problem which is related to Ray's theorem [13] in the geometry of Banach spaces; see also [4, 12]. Kocourek, Takahashi and Yao [5] defined a broad class of nonlinear mappings which contains nonexpansive mappings, nonspreading mappings and hybrid mappings in a Hilbert space. A mapping $T : C \to H$ is called *generalized hybrid* [5] if there exist $\alpha, \beta \in \mathbb{R}$ such that

$$\alpha\|Tx - Ty\|^2 + (1 - \alpha)\|x - Ty\|^2 \leq \beta\|Tx - y\|^2 + (1 - \beta)\|x - y\|^2$$

for all $x, y \in C$, where $\mathbb{R}$ is the set of real numbers. A mapping T is called an (α, β)-*generalized hybrid* mapping; see also [2]. Kocourek, Takahashi and Yao [5] proved fixed point theorems for such mappings in a Hilbert space. See also [6, 18]. They also proved the following mean convergence theorem which generalizes Baillon's nonlinear ergodic theorem [3] in a Hilbert space. See also [9, 10, 14, 15].

Theorem 1 ([5]) *Let H be a real Hilbert space, let C be a nonempty, closed and convex subset of H, let T be a generalized hybrid mapping from C into itself with $F(T) \neq \emptyset$ and let P be the metric projection of H onto $F(T)$. Then for any $x \in C$, $S_n x = \frac{1}{n}\sum_{k=0}^{n-1} T^k x$ converges weakly to $p \in F(T)$, where $p = \lim_{n\to\infty} PT^n x$.*

Takahashi and Takeuchi [17] introduced the concept of attractive points of mappings in a Hilbert space and they proved attractive point and mean convergence theorems without convexity for generalized hybrid mappings; see also [1].

In this talk, we introduce the concept of attractive points and adjoint operators of nonlinear mappings in Banach spaes. We prove attractive point theorems, duality theorems and mean convergence theorems for generalized nonspreading mappings in Banach spaces.

[1] S. Akashi and W. Takahashi, *Strong convergence theorem for nonexpansive mappings on star-shaped sets in Hilbert spaces*, Appl. Math. Comput., to appear.

[2] K. Aoyama, S. Iemoto, F. Kohsaka and W. Takahashi, *Fixed point and ergodic theorems for λ-hybrid mappings in Hilbert spaces*, J. Nonlinear Convex Anal. **11** (2010), 335-343.

[3] J.-B. Baillon, *Un theoreme de type ergodique pour les contractions non lineaires dans un espace de Hilbert*, C. R. Acad. Sci. Paris Ser. A-B **280** (1975), 1511–1514.

[4] S. Kamimura and W. Takahashi, *Strong convergence of a proximal-type algorithm in a Banach space*, SIAM J. Optim. **13** (2002), 938–945.

[5] P. Kocourek, W. Takahashi and J.-C. Yao, *Fixed point theorems and weak convergence theorems for generalized hybrid mappings in Hilbert spaces*, Taiwanese J. Math. **14** (2010), 2497–2511.

[6] P. Kocourek, W. Takahashi and J.-C. Yao, *Fixed point theorems and ergodic theorems for nonlinear mappings in Banach spaces*, Adv. Math. Econ. **15** (2011), 67–88.

[7] F. Kohsaka and W. Takahashi, *Existence and approximation of fixed points of firmly nonexpansive-type mappings in Banach spaces*, SIAM J. Optim. **19** (2008), 824–835.

[8] F. Kohsaka and W. Takahashi, *Fixed point theorems for a class of nonlinear mappings related to maximal monotone operators in Banach spaces*, Arch. Math. (Basel) **91** (2008), 166-177.

[9] A. T. Lau and W. Takahashi, *Invariant submeans and semigroups of nonexpansive mappings on Banach spaces with normal structure*, J. Funct. Anal. **142** (1996), 79-88.

[10] A. T. Lau, N. Shioji and W. Takahashi, *Existence of nonexpansive retractions for amenable semigroups of nonexpansive mappings and nonlinear ergodic theorems in Banach spaces*, J. Funct. Anal. **161** (1999), 62-75.

[11] L.-J. Lin and W. Takahashi, *Attractive point theorems for nonspreading mappings in Banach space*, J. Convex Anal., to appear.

[12] N. Nadezhkina and W. Takahashi, *Strong convergence theorem by a hybrid method for nonexpansive mappings and Lipschitz-continuous monotone mappings*, SIAM J. Optim. **16**, (2006), 1230–1241.

[13] W. O. Ray, *The fixed point property and unbounded sets in Hilbert space*, Trans. Amer. Math. Soc. **258**, (1980), 531–537.

[14] W. Takahashi, *A nonlinear ergodic theorem for an amenable semigroup of nonexpansive mappings in a Hilbert space*, Proc. Amer. Math. Soc. **81**, (1981), 253–256.

[15] W. Takahashi, *A nonlinear ergodic theorem for a reversible semigroup of nonexpansive mappings in a Hilbert space*, Proc. Amer. Math. Soc. **96**, (1986), 55-58.

[16] W. Takahashi, *Fixed point theorems for new nonlinear mappings in a Hilbert space*, J. Nonlinear Convex Anal. **11**, (2010), 79–88.

[17] W. Takahashi and Y. Takeuchi, *Nonlinear ergodic theorem without convexity for generalized hybrid mappings in a Hilbert space*, J. Nonlinear Convex Anal. **12**, (2011), 399–406.

[18] W. Takahashi, N.-C. Wong and J.-C. Yao, *Fixed point theorems and convergence theorems for generalized nonspreading mappings in Banach spaces*, J. Fixed Point Theory Appl. **11**, (2012), 159–183.

[19] W. Takahashi, N.-C. Wong and J.-C. Yao, *Attractive point and mean convergence theorems for new generalized nonspreading mappings in Banach Spaces*, to appear.

[20] W. Takahashi, J. C. Yao and F. Kohsaka, *The fixed point property and unbounded sets in Banach spaces*, Taiwanese J. Math. **14** (2010), 733–742.

A new sequence space $\Lambda_p(f)$

Yoshiaki Okazaki

Kyushu Institute of Technology,
680-4, Kawazu, Iizuka, Fukuoka 820-8502, Japan
Email **okazaki @ces.kyutech.ac.jp**

This talk is a joint work with Prof. Aoi Honda (Kyushu Institute of Technology) and Emer. Prof. Hiroshi Sato (Kyushu University).

For $f(\neq 0) \in L_p(\mathbb{R}, dx), 1 \leq p < +\infty$, a sequence space $\Lambda_p(f)$ and a translation invariant metric d_p^f are defined by

$$\begin{aligned}\Lambda_p(f) &:= \left\{ \boldsymbol{a} = \{a_k\} \in \mathbb{R}^\infty \,\middle|\, \sum_{k=1}^{+\infty} \int_{-\infty}^{+\infty} |f(x-a_k) - f(x)|^p dx < +\infty \right\}, \\ d_p^f(\boldsymbol{a}, \boldsymbol{b}) &:= \left(\sum_k \int_{-\infty}^{+\infty} |f(x-a_k) - f(x-b_k)|^p dx \right)^{\frac{1}{p}}, \ \boldsymbol{a}, \boldsymbol{b} \in \Lambda_p(f).\end{aligned}$$

$\Lambda_p(f)$ is included in ℓ_p and realizes various sequence spaces such as the Zygmund type spaces. For example, $\Lambda_2(\sqrt{x}e^{-x}\mathbf{1}_{[0,+\infty)}(x)) = \ell_2(\log \ell)$.

In the case $p > 1$, $\Lambda_p(f) = \ell_p$ if and only if f is absolutely continuous and $I_p(f) := \int_{\mathbb{R}} |f'(x)|^p dx < +\infty$. For $f, g \in L_p(\mathbb{R}, dx)$, if $I_p(f-g) < +\infty$, then we have $\Lambda_p(f) = \Lambda_p(g)$. Furthermore for $f \in L_p(\mathbb{R}, dx),\ g \in L_r(\mathbb{R}, dx), 1 \leq p,\ r < +\infty$, we have $\Lambda_p(f) \subset \Lambda_r(g)$ if and only if the embedding $\Lambda_p(f) \to \Lambda_r(g)$ is continuous.

$\Lambda_p(f)$ is not necessarily a linear space, and its explicit representation as a sequence space is not clear. In this talk, in the case $p = 2$, we discuss these problems by utilizing the Fourier transform $\hat{f}$ of f and a function $\varphi_f(x) := \int_0^x \alpha^2 |\hat{f}(\alpha)|^2 d\alpha$. We introduce approximation spaces $\Lambda_2^0(f) \subset \Lambda_2(f) \subset \Lambda_2^\varphi(f)$ by

$$\begin{aligned}\Lambda_2^0(f) &:= \left\{ \boldsymbol{a} \in \mathbb{R}^\infty \,\middle|\, \sum_k a_k^2 \varphi_f\left(\frac{1}{|a_k|}\right) + \sum_k \int_{\frac{1}{|a_k|}}^{+\infty} |\hat{f}(\alpha)|^2 d\alpha < +\infty \right\}, \\ \Lambda_2^\varphi(f) &:= \left\{ \boldsymbol{a} \in \mathbb{R}^\infty \,\middle|\, \sum_k a_k^2 \left[1 + \varphi_f\left(\frac{1}{|a_k|}\right)\right] < +\infty \right\}.\end{aligned}$$

$\Lambda_2^0(f)$ is the maximum linear subspace of $\Lambda_2(f)$. We define the *doubling dimension* $H(\varphi)$ for a non-negative function φ and prove if $H(\varphi_f) < 2$ then we have $\Lambda_2^0(f) = \Lambda_2(f) = \Lambda_2^\varphi(f)$, so that $\Lambda_2(f)$ is a linear space, and d_2^f is equivalent to a quasi-metric $\delta_f(\boldsymbol{a}, \boldsymbol{b}) := \sqrt{\sum_k |a_k - b_k|^2 \wedge 1 \left[1 + \varphi_f\left(\frac{1}{|a_k - b_k| \wedge 1}\right)\right]}$.

Moduli of smoothness related to the Laplacian and approximation by Bochner-Riesz families

Hans-Jürgen Schmeisser

Mathematical Institute, Friedrich-Schiller-University
D-07737 Jena, Germany
Email **mhj @ uni-jena.de**

Let $\mathcal{T}_n,\ n \in \mathbb{N}$, be the space of all real-valued trigonometric polynomials of (spherical) order at most n and let $f \in L_p(\mathbb{T}^d),\ 0 < p < \infty$. The polynomial $\mathcal{K}$-functional related to the Laplacian is defined as

$$\mathcal{P}_\Delta(f, n^{-1})_p \;=\; \inf\{\|f - g\|_p + n^{-2}\|\Delta g\|_p \;:\; g \in \mathcal{T}_n\;\}. \tag{1}$$

Following [1] - [3] we discuss the equialence of (1) and associated moduli of smoothness. We focus on the case $0 < p < 1$ and introduce new moduli of smoothness

$$\omega_{m,d}(f,\delta)_p = \sup_{0 \le h \le \delta} \left\| \frac{\sigma_m}{d} \sum_{j=1}^{d} \sum_{\substack{\nu = -m \\ \nu \neq 0}}^{m} \frac{(-1)^\nu}{\nu^2} \binom{2m}{m - |\,\nu\,|} f(x + \nu h e_j) - f(x) \right\|_p , \tag{2}$$

where σ_m is an appropriate constant, m is a natural number and e_j stands for the unit vector with respect to the j-th coordinate. Our main result is

Theorem 1 *Let $m,\ d \in \mathbb{N}$, $d > 2$. If $\dfrac{d}{d+2m} < p < \infty$ then*

$$\mathcal{P}_\Delta(f, n^{-1})_p \asymp \omega_{m,d}(f, n^{-1})_p\,,\ n \in \mathbb{N}\ \ f \in L_p(\mathbb{T}^d). \tag{3}$$

Moreover, we show that quantities (1) and (2) are associated with approximation by Bochner-Riesz families in a natural way.
This is joint work with K. Runovski (Sevastopol).

[1] Ditzian, Z. and Runovski, K: *Realization and smoothness related to the Laplacian.* Acta Math Hungar. **99 (3)** (2001), 189–223.

[2] Rukasov, V., Runovski, K. and H.-J. Schmeisser: *Approximation by families of linear trigonometric polynomial operators and smoothness properties of functions.* Math. Nachr. **284 (11-12)** (2011), 1523–1537.

[3] K. Runovski and H.-J. Schmeisser: *On approximation methods generated by Bochner-Riesz kernels.* J. Fourier Anal. Appl. **14** (2008), 16–38.

Cesàro and Riesz means of critical order on certain function spaces

Shuichi Sato

Department of Mathematics, Faculty of Education,
Kanazawa University, Kanazawa 920-1192, Japan
Email **shuichi @kenroku.kanazawa-u.ac.jp**

Let f be in $L^1(Q_d)$, where Q_d is the fundamental cube in the d-dimensional Euclidean space $\mathbb{R}^d$:

$$Q_d = \{x \in \mathbb{R}^d : -1/2 < x_i \leq 1/2, i = 1, 2, \ldots, d\}, \quad x = (x_1, \ldots, x_d).$$

We consider the Fourier series of f:

$$f(x) \sim \sum a_n e^{2\pi i \langle n,x \rangle}, \quad n = (n_1, n_2, \ldots, n_d) \in \mathbb{Z}^d,$$
$$\langle n, x\rangle = n_1 x_1 + \cdots + n_d x_d,$$
$$a_n = \int_{Q_d} f(x) e^{-2\pi i \langle n,x\rangle}\, dx, \quad dx = dx_1 \ldots dx_d.$$

Let

$$T_R^\delta(f)(x) = \sum_{|n|<R} \left(1 - \frac{|n|^2}{R^2}\right)^\delta a_n e^{2\pi i\langle n,x\rangle}$$

be the Bochner-Riesz means of order δ of the series.

We define a logconvex quasi-Banach space $\mathfrak{Q}\mathcal{A}(Q_d)$. We say $f \in \mathfrak{Q}\mathcal{A}(Q_d)$ if there exists a sequence $\{f_j\}$ of bounded functions such that

$$f = \sum_{j=1}^\infty f_j, \quad N(\{f_j\}) := \sum_{j=1}^\infty (1 + \log j)\|f_j\|_1 \log\left(\frac{e\|f_j\|_\infty}{\|f_j\|_1}\right) < \infty;$$

let $\|f\|_{\mathfrak{Q}\mathcal{A}} = \inf N(\{f_j\})$, where the infimum is taken over all possible $\{f_j\}$. The space $\mathfrak{Q}\mathcal{A}(Q_1)$ was introduced by [2]. $\mathfrak{Q}\mathcal{A}$ is a subspace of $L \log L$.

Define $T_*^\delta(f)(x) = \sup_{R>0} |T_R^\delta(f)(x)|$.

Theorem 1 *Let $\alpha = (d-1)/2$. Then*

$$\|T_*^\alpha(f)\|_{1,\infty} = \sup_{\lambda>0} \lambda |\{x \in Q_d : T_*^\alpha(f)(x) > \lambda\}| \leq C\|f\|_{\mathfrak{Q}\mathcal{A}}$$

with a positive constant C; consequently,

$$\lim_{R\to\infty} T_R^\alpha(f)(x) = f(x) \quad a.e. \qquad for\ f \in \mathfrak{Q}\mathcal{A}(Q_d).$$

Since $L \log L \log\log\log L$ is a proper subspace of $\mathfrak{Q}\mathcal{A}$, Theorem 1 implies the following.

Theorem 2 *Let $f \in L \log L \log\log\log L(Q_d)$. Then*

$$\lim_{R\to\infty} T_R^\alpha(f)(x) = f(x) \quad a.e.$$

We have similar results for the Cesàro means of spherical harmonics expansions. Let $\Sigma_d = \{x \in \mathbb{R}^{d+1} : |x| = 1\}$ be the unit sphere in $\mathbb{R}^{d+1}$ and let

$$H_k f(x) = \int_{\Sigma_d} Z_x^{(k)}(y) f(y)\, d\mu(y),$$

where $Z_x^{(k)}$ is the zonal harmonic of degree k with pole $x \in \Sigma_d$ and $d\mu$ is the Lebesgue surface measure on Σ_d normalized as $\mu(\Sigma_d) = 1$. The Cesàro means of order δ for the spherical harmonics expansion $f \sim \sum_{k=0}^\infty H_k f$ are defined by

$$S_n^\delta f = \frac{1}{A_n^{(\delta)}} \sum_{k=0}^{n} A_{n-k}^{(\delta)} H_k f, \quad n = 0, 1, 2, \dots, \quad \delta = \sigma + i\tau,$$
$$A_k^{(\delta)} = \frac{\Gamma(k+\delta+1)}{\Gamma(k+1)\Gamma(\delta+1)} = \binom{k+\delta}{k}, \quad \sigma > -1.$$

Let $S_*^\delta(f)(x) = \sup_{n>0} |S_n^\delta(f)(x)|$. We define the space $\mathfrak{Q}\mathcal{A}(\Sigma_d)$ analogously to $\mathfrak{Q}\mathcal{A}(Q_d)$. We write $|E| = \mu(E)$ for $E \subset \Sigma_d$.

Theorem 3 *There exists a positive constant C such that*

$$\sup_{\lambda>0} \lambda |\{x \in \Sigma_2 : S_*^{1/2}(f)(x) > \lambda\}| \le C\|f\|_{\mathfrak{Q}\mathcal{A}}$$

for $f \in \mathfrak{Q}\mathcal{A}(\Sigma_2)$, which implies

$$\lim_{n\to\infty} S_n^{1/2}(f)(x) = f(x) \quad a.e. \qquad \text{for } f \in \mathfrak{Q}\mathcal{A}(\Sigma_2).$$

Theorem 4 *If $f \in L \log L \log\log\log L(\Sigma_2)$, then*

$$\lim_{n\to\infty} S_n^{1/2} f(x) = f(x) \quad a.e.$$

[1] N. Yu. Antonov, *Convergence of Fourier series*, East J. Approx. **2** (1996), 187–196.

[2] J. Arias-de-Reyna, *Pointwise convergence of Fourier series*, J. London Math Soc. (2) **65** (2002), 139–153.

[3] R. Askey and I. I. Hirschiman, *Mean summability for ultraspherical polynomials*, Math. Scand. **12** (1963), 167–177.

[4] A. Bonami and J.-L. Clerc, *Sommes de Cesàro et multiplicateurs des développments en harmonique sphérique*, Trans. Amer. Math. Soc. **183** (1973), 223–263.

[5] L. Carleson, *On convergence and growth of partial sums of Fourier series*, Acta Math. **116** (1966), 135–157.

[6] L. Colzani, M. H. Taibleson and G. Weiss, *Maximal estimates for Cesàro and Riesz means on spheres*, Indiana Univ. Math. J. **33** (1984), 873–889.

[7] C. Fefferman, *Pointwise convergence of Fourier series*, Ann. of Math. **98** (1973), 551–572.

[8] R. Hunt, *On the convergence of Fourier series*, Orthogonal Expansions and their Continuous Analogues (Edwardsville, IL, 1967), Southern Illinois Univ. Press, 1968, 235–255.

[9] N. J. Kalton, *Convexity type and the three space problem*, Studia Math. **69** (1981), 247–287.

[10] E. Kogbetliantz, *Recherches sur la sommabilité des séries ultrasphériques par la méthode des moyennes arithmétiques*, J. Math. Pures Appl. **3** (1924), 107–187.

[11] Y. Sagher, *On analytic families of operators*, Israel Journ. Math. **7** (1969), 350–356.

[12] P. Sjölin, *An inequality of Paley and convergence a. e. of Walsh-Fourier series*, Ark. Mat. **7** (1968), 551–570.

[13] P. Sjölin and F. Soria, *Remarks on a theorem by N. Yu. Antonov*, Studia Math. **158** (2003), 79–97.

[14] C. D. Sogge, *Oscillatory integrals and spherical harmonics*, Duke Math. J. **53** (1986), 43–65.

[15] E. M. Stein, *Localization and summability of multiple Fourier series*, Acta Math. **100** (1958), 93–147.

[16] E. M. Stein and G. Weiss, *Fourier Analysis on Euclidean Spaces*, Princeton Univ. Press, 1971.

[17] G. Sunouchi, *On the summability almost everywhere of the multiple Fourier series at the critical index*, Kodai Math. J. **8** (1985), 1–4.

[18] G. Szegö, *Orthogonal Polynomials*, 4th ed., Amer Math. Soc. Coll. Publ. No. 23, Providence, 1975.

[19] A. Zygmund, *Trigonometric Series*, 2nd ed., Cambridge Univ. Press, Cambridge, London, New York and Melbourne, 1977.

Sep–12 1500–1530

Segal algebras which are neither BSE nor BED

Sin-Ei Takahasi

Toho University, Japan
Email **sin_ei1 @yahoo.co.jp**

Let A be the Fourier algebra on a non-compact LCA group. We show that there are proper Segal algebra in A which are not contained in any proper isometrically translation invariant Segal algebras in A. This result is used to construct a class of Segal algebras which are neither BSE nor BED.

Sobolev's inequality for Riesz potentials of functions in grand Morrey spaces of variable exponent

Yoshihiro Mizuta

Department of Mechanical Systems Engineering
Hiroshima Institute of Technology
2-1-1 Miyake Saeki-ku Hiroshima 731-5193, Japan
Email **y.mizuta.5x @ it-hiroshima.ac.jp**

Let $\mathbf{R}^N$ denote the N-dimensional Euclidean space. We denote by $B(x,r)$ the open ball centered at x of radius r and denote by $|E|$ the Lebesgue measure of a measurable set $E \subset \mathbf{R}^N$. In our discussions, the boundedness of the Hardy-Littlewood maximal operator is a crucial tool as in Hedberg [15]. It is well known that the maximal operator is bounded on the Lebesgue space $L^p(\mathbf{R}^N)$ if $p > 1$ (see [28]).

In 1938, Morrey [22] considered the integral growth condition on derivatives over balls, in order to study the existence and regularity for partial differential equations. A family of functions with the integral growth condition is then called a Morrey space after his name. A systematical study for Morrey spaces was done by Peetre [24] in 1969. Chiarenza-Frasca [4] generalized the boundedness of the maximal operator by replacing Lebesgue spaces bythe Morrey space $L^{p,\nu}(\mathbf{R}^N)$, which is the family of $f \in L^1_{loc}(\mathbf{R}^N)$ satisfying the Morrey condition

$$\sup_{x \in \mathbf{R}^N, r>0} \frac{r^\nu}{|B(x,r)|} \int_{B(x,r)} |f(y)|^p dy < \infty$$

for $\nu > 0$ (see also Nakai [23]).

In [6], Diening showed that the maximal operator was bounded on the variable exponent Lebesgue space $L^{p(\cdot)}(\mathbf{R}^N)$ if the variable exponent $p(\cdot)$ is a constant outside a ball, it satisfies the local log-Hölder condition and $\inf p(x) > 1$. In the mean time, variable exponent Lebesgue spaces and Sobolev spaces were introduced to discuss nonlinear partial differential equations with non-standard growth condition. These spaces have attracted more and more attention, in connection with the study of elasticity, fluid mechanics; see [25]. In the case of bounded open sets, Almeida-Hasanov-Samko [2], Guliyev-Hasanov-Samko [13, 14] and Mizuta-Shimomura [21] studied the boundedness of the maximal operator for the variable exponent Morrey spaces.

Grand Lebesgue spaces were introduced in [16] for the sake of study of the integrability of the Jacobian. Grand Lebesgue spaces have been considered in various fields: in the theory of partial differential equations (see e.g. [17, 18, 26, 27]) and in the study of maximal operators (see e.g. [9]). In particular, in the theory of partial differential equations, it turns out that they are the right spaces in which N-harmonic equations $\mathrm{div}(|\nabla u|^{N-2}\nabla u) = \mu$ have to be considered (see [10, 12]). Further they have been studied in their own (see e.g. [3, 11]). Fiorenza-Gupta-Jain [8] studied the boundedness of the maximal operator in the grand Lebesgue spaces $L^{p)}([0,1])$ (see also [19]). Meskhi [20] generalized the boundedness of the maximal operator by replacing grand Lebesgue spaces by grand Morrey spaces $L^{p),\nu,\theta}(G)$, which is the family of $f \in L^1_{loc}(G)$ satisfying the grand Morrey condition

$$\sup_{x\in G, 0<r<d_G, 0<\varepsilon<p-1} \varepsilon^\theta \frac{r^\nu}{|B(x,r)|} \int_{B(x,r)} |f(y)|^{p-\varepsilon}\, dy < \infty,$$

where G is a bounded open set in $\mathbf{R}^N$, $\nu > 0$ and $\theta > 0$.

Our first aim in this talk is to establish the boundedness of the maximal operator in grand Morrey spaces of variable exponents, as an extension of Meskhi [20].

For $0 < \alpha < N$ and a locally integrable function f on G, we define the Riesz potential $U_\alpha f$ of order α by

$$U_\alpha f(x) = \int_G |x-y|^{\alpha-N} f(y)\, dy.$$

One of important applications of the boundedness of the maximal operator is Sobolev's inequality; in the classical case,

$$\|U_\alpha f\|_{L^{p^*}(\mathbf{R}^N)} \le C\|f\|_{L^p(\mathbf{R}^N)}$$

for $f \in L^p(\mathbf{R}^N)$, $0 < \alpha < N$ and $1 < p < N/\alpha$. Sobolev's inequality has been studied in many articles and settings. If $f \in L^{p,\nu}(\mathbf{R}^N)$, then it is shown (see Adams [1] and Peetre [24]) that $U_\alpha f$ satisfies Sobolev's inequality whenever $\nu > \alpha p$, where $1 < p < \infty$. Diening [7] dealt with Sobolev's embeddings for Riesz potentials with functions in $L^{p(\cdot)}(\mathbf{R}^N)$. In the case of bounded open sets, Almeida-Hasanov-Samko [2] and Mizuta-Shimomura [21] have established embedding results for Riesz potentials of functions in the variable exponent Morrey spaces. The version for the generalized variable exponent Morrey space $L^{p(\cdot),\omega}(G)$ was discussed by Guliyev-Hasanov-Samko [13, 14]. Further, Meskhi [20] studied Sobolev's embeddings for Riesz potentials of functions in the grand Morrey spaces.

Our second aim in this talk, as an application of the boundedness of maximal operator, is to establish Sobolev type inequalities for Riesz potentials of functions in grand Morrey spaces of variable exponents, as an extension of Meskhi [20].

[1] D. R. Adams, A note on Riesz potentials, Duke Math. J. **42** (1975), 765–778.

[2] A. Almeida, J. Hasanov and S. Samko, Maximal and potential operators in variable exponent Morrey spaces, Georgian Math. J. **15** (2008), 195-208.

[3] M. Carozza and C. Sbordone, The distance to L^∞ in some function spaces and applications, Differential Integral Equations, **10** (1997), 599–607.

[4] F. Chiarenza and M. Frasca, Morrey spaces and Hardy-Littlewood maximal function, Rend. Mat. Apple. **7**(7) (1987), 273–279.

[5] D. Cruz-Uribe, A. Fiorenza and C. J. Neugebauer, The maximal function on variable L^p spaces, Ann. Acad. Sci. Fenn. Ser. Math. **28** (2003), 223–238, **29** (2004), 247–249.

[6] L. Diening, Maximal functions in generalized $L^{p(\cdot)}$ spaces, Math. Inequal. Appl. **7**(2) (2004), 245–254.

[7] L. Diening, Riesz potentials and Sobolev embeddings on generalized Lebesgue and Sobolev spaces $L^{p(\cdot)}$ and $W^{k,p(\cdot)}$, Math. Nachr. **263**(1) (2004), 31–43.

[8] A. Fiorenza, B. Gupta and P. Jain, The maximal theorem in weighted grand Lebesgue spaces, Stud. Math. **188**(2) (2008), 123–133.

[9] A. Fiorenza and M. Krbec, On the domain and range of the maximal operator, Nagoya Math. J. **158** (2000), 43–61.

[10] A. Fiorenza and C. Sbordone, Existence and uniqueness results for solutions of nonlinear equations with right hand side in L^1, Studia Math. **127**(3) (1998), 223–231.

[11] L. Greco, A remark on the equality $\mathrm{det}Df = \mathrm{Det}Df$, Differential Integral Equations **6** (1993), 1089–1100.

[12] L. Greco, T. Iwaniec and C. Sbordone, Inverting the p-harmonic operator, Manuscripta Math. **92** (1997), 249–258.

[13] V. S. Guliyev, J. Hasanov and S. Samko, Boundedness of the maximal, potential and singular operators in the generalized variable exponent Morrey spaces, Math. Scand. **107** (2010), 285–304.

[14] V. S. Guliyev, J. Hasanov and S. Samko, Boundedness of the maximal, potential and Singular integral operators in the generalized variable exponent Morrey type spaces, Journal of Mathematical Sciences **170** (2010), no. 4, 423–443.

[15] L. I. Hedberg, On certain convolution inequalities, Proc. Amer. Math. Soc. **36** (1972), 505–510.

[16] T. Iwaniec and C. Sbordone, On the integrability of the Jacobian under minimal hypotheses, Arch. Rational Mech. Anal. **119** (1992), 129–143.

[17] T. Iwaniec and C. Sbordone, Weak minima of variational integrals, J. Reine Angew. Math. **454** (1994), 143–161.

[18] T. Iwaniec and C. Sbordone, Riesz Transforms and elliptic pde's with VMO coefficients, J. Analyse Math. **74** (1998), 183–212.

[19] V. Kokilashvili and S. Samko, Boundedness of weighted singular integral operators in grand Lebesgue spaces to appear in Georgian Math. J.

[20] A. Meskhi, Maximal functions and singular integrals in Morrey spaces associated with grand Lebesgue spaces. Proc. A. Razmadze Math. Inst. **151** (2009), 139–143.

[21] Y. Mizuta and T. Shimomura, Sobolev embeddings for Riesz potentials of functions in Morrey spaces of variable exponent, J. Math. Soc. Japan **60** (2008), 583-602.

[22] C. B. Morrey, On the solutions of quasi-linear elliptic partial differential equations, Trans. Amer. Math. Soc. **43** (1938), 126–166.

[23] E. Nakai, Hardy-Littlewood maximal operator, singular integral operators and the Riesz potentials on generalized Morrey spaces, Math. Nachr. **166** (1994), 95–103.

[24] J. Peetre, On the theory of $L_{p,\lambda}$ spaces, J. Funct. Anal. **4** (1969), 71-87.

[25] M. Růžička, *Electrorheological Fluids: Modeling and Mathematical Theory*, Springer-Verlag, Berlin, 2000.

[26] C. Sbordone, Grand Sobolev spaces and their applications to variational problems, Le Matematiche **LI**(2) (1996), 335–347.

[27] C. Sbordone, Nonlinear elliptic equations with right hand side in nonstandard spaces, Rend. Sem. Mat. Fis. Modena, Supplemento al **XLVI** (1998), 361–368.

[28] E. M. Stein, *Singular integrals and differentiability properties of functions*, Princeton Univ. Press, Princeton, 1970.

Sep–12 1610–1630

On the existence of solutions of second order ordinary differential equations

Toshiharu Kawasaki and Masashi Toyoda*

College of Engineering, Nihon University, Fukushima 963–8642, Japan
Email **toshiharu.kawasaki @ nifty.ne.jp**

Faculty of Engineering, Tamagawa University, Tokyo 194–8610, Japan
Email **mss-toyoda @ eng.tamagawa.ac.jp**

In [4], Knežević-Miljanović considered the Cauchy problem

$$\begin{cases} u''(t) = P(t)t^a u(t)^\sigma, \ t \in (0,1], \\ u(0) = 0, \ u'(0) = \lambda, \end{cases} \tag{1}$$

where P is continuous, $a, \sigma, \lambda \in \mathbf{R}$ with $\sigma < 0$ and $\lambda > 0$, and $\int_0^1 |P(t)|t^{a+\sigma}dt < \infty$. In [3], Kawasaki and Toyoda considered the Cauchy problem

$$\begin{cases} u''(t) = f(t, u(t)), \ \text{for almost every} \ \ t \in [0,1], \\ u(0) = 0, \ u'(0) = \lambda, \end{cases} \tag{2}$$

where f is a mapping from $[0,1] \times (0,\infty)$ into $\mathbf{R}$ and $\lambda \in \mathbf{R}$ with $\lambda > 0$. They proved the unique solvability of the Cauchy problem (2) using the Banach fixed point theorem. The theorem in [3] is the following.

Theorem 1 *Suppose that a mapping f from $[0,1] \times [0,\infty)$ into $\mathbf{R}$ satisfies the following.*

1. *The mapping $t \longmapsto f(t,u)$ is measurable for any $u \in (0,\infty)$ and the mapping $u \longmapsto f(t,u)$ is continuous for almost every $t \in [0,1]$.*
2. *$|f(t,u_1)| \geq |f(t,u_2)|$ for almost every $t \in [0,1]$ and for any $u_1, u_2 \in [0,\infty)$ with $u_1 \leq u_2$.*
3. *There exists $\alpha \in \mathbf{R}$ with $0 < \alpha < \lambda$ such that*
$$\int_0^1 |f(t,\alpha t)|dt < \infty.$$
4. *There exists $\beta \in \mathbf{R}$ with $\beta > 0$ such that*
$$\left| \frac{\partial f}{\partial u}(t,u) \right| \leq \frac{\beta |f(t,u)|}{u}$$
for almost every $t \in [0,1]$ and for any $u \in (0,\infty)$.

Then there exist $h \in \mathbf{R}$ *with* $0 < h \leq 1$ *such that the Cauchy problem* (2) *has a unique solution in* X*, where* X *is a subset* $X = \{u \mid u \in C[0,h], u(0) = 0, u'(0) = \lambda$ *and* $\alpha t \leq u(t)$ *for any* $t \in [0,h]\}$.

The case that $f(t, u(t)) = P(t)t^a u(t)^\sigma$ is the theorem of Knežević-Miljanović [4]. Moreover, we can prove the unique solvability of the problem

$$\begin{cases} u''(t) = f(t, u(t), u'(t)), \text{ for almost every } t \in [0,1], \\ u(0) = 0, \; u'(0) = \lambda, \end{cases}$$

where f is a mapping from $[0,1] \times (0,\infty) \times \mathbf{R}$ into $\mathbf{R}$ and $\lambda \in \mathbf{R}$ with $\lambda > 0$.

[1] L. H. Erbe and H. Wang, *On the existence of positive solutions of ordinary differential equations*, Proceedings of the American Mathematical Society, 120 (1994), 743–748.

[2] H. T. Davis, *Introduction to Nonlinear Differential and Integral Equations*, Dover Publications, New York (1962).

[3] T. Kawasaki and M. Toyoda, *Existence of positive solution for the Cauchy problem for an ordinary differential equation*, Nonlinear Mathematics for Uncertainly and its Applications, Advances in Intelligent and Soft Computing, 100 (2011), Berlin and New York, 435–441.

[4] J. Knežević-Miljanović, *On the Cauchy problem for an Emden-Fowler equation*, Differential Equations 45 (2009), 267–270.

[5] J. S. W. Wong, *On the generalized Emden-Fowler equation*, SIAM Review 17 (1975), 339–360.

Sep–12 1630–1650

Boundedness of integral operators in weighted Sobolev space

Oinarov Ryskul

Department of Mathematics, L.N. Gumilyov Eurasian National University,
5 Munaytpasov St, 010008 Astana, Kazakhstan
Email **o_ryskul @ mail.ru**

For the integral operator $Kf(x) = \int\limits_a^x K(x,t)f(t)dt$, $x \in (a,b) \subseteq R$, under some assumptions on the kernel $K(x,t) \geq 0$ we find necessary and sufficient conditions for the validity of the following weighted inequalities

$$\|wKf\|_q \leq C\left(\|\rho f'\|_p + \|vf\|_p\right), \quad \forall f \in A\overset{\circ}{C}\,(a,b),$$

and

$$\|u\frac{d}{dx}Kf\|_q \leq C\left(\|\rho f'\|_p + \|vf\|_p\right), \quad \forall f \in A\overset{\circ}{C}\,(a,b),$$

where w, u, ρ and v are weighted functions, $A\overset{\circ}{C}(a,b)$ is a set of absolutely continuous functions with compact supports.

Sep–12 1700–1720

Strong convergence of an iterative scheme for two different types of operators

Yasunori Kimura* and Satit Saejung

Department of Information Science, Faculty of Science, Toho University,
Miyama, Funabashi, Chiba 274-8510, Japan
Email **yasunori @is.sci.toho-u.ac.jp**

Department of Mathematics, Faculty of Science, Khon Kaen University,
Khon Kaen 40002, Thailand
Email **saejung @kku.ac.th**

We deal with the problem of finding a common fixed point for nonlinear mappings, which is one of the most important problems in nonlinear analysis and has been studied by a large number of researchers for various types of nonlinear mappings.

In this talk, we will propose iterative schemes for two different types of nonlinear mappings defined on a Banach space, whose generated sequence converges strongly to their common fixed point.

Sep–12 1720–1740

Some generalized asymptotic pointwise ρ-contraction mappings involving orbits in Modular spaces

Poom Kumam* and Chirasak Mongkolkeha

Department of Mathematics, Faculty of Science,
King Mongkut's University of Technology Thonburi (KMUTT),
Bangmod, Bangkok 10140, Thailand
Email **poom.kum @kmutt.ac.th**

Department of Mathematics, Faculty of Science,
King Mongkut's University of Technology Thonburi (KMUTT),
Bangmod, Bangkok 10140, Thailand
Email **cm.mongkol @hotmail.com**

In this paper, we prove new fixed point theorems for generalized asymptotic pointwise $\rho-$contractions and pointwise asymptotically $\rho-$ nonexpansive using the radius of the orbit in modular spaces. The results of this paper we improve and extend results of Nicolae [1] and Kuaket and Kumam [2].

[1] A. Nicolae, Generalized asymptotic pointwise contractions and nonexpansive mappings involving orbits, Fixed Point Theory and Applications, (2009) Article ID 458265, 19 pages

[2] K. Kuaket, P. Kumam, Fixed point for asymptotic point5wise contractions in modular spaces, Applied Mathematics Letters, (2011), 24 (11), 1795-1798.

Sep–12 1740–1800

Fixed point theorems for some nonlinear mappings related to resolvents of monotone or accretive operators

Fumiaki Kohsaka

Department of Computer Science and Intelligent Systems, Oita University,
700 Dannoharu, Oita-shi, Oita 870-1192, Japan
Email **f-kohsaka @oita-u.ac.jp**

The problem of solving the operator inclusion $Au \ni 0$ for some maximal monotone operator $A\colon H \to 2^H$ in a real Hilbert space H is a fixed point problem for some firmly nonexpansive mapping.

Let C be a nonempty subset of a real Hilbert space H and $T\colon C \to H$ a mapping. Recall the following definitions: T is said to be nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$; T is said to be firmly nonexpansive if $\|Tx - Ty\|^2 \leq \langle Tx - Ty, x - y\rangle$ for all $x, y \in C$; see [3, 4]; T is said to be nonspreading if $2\|Tx-Ty\|^2 \leq \|Tx-y\|^2+\|Ty-x\|^2$ for all $x, y \in C$; see [6]. It is known that every firmly nonexpansive mapping is both nonexpansive and nonspreading.

In this talk, we obtain some generalizations of the following fixed point theorem, which was originally shown in Banach spaces.

Theorem 1 ([6]) *Every nonspreading self mapping of a nonempty bounded closed convex subset of a real Hilbert space has a fixed point.*

[1] K. Aoyama and F. Kohsaka, *Fixed point theorem for α-nonexpansive mappings in Banach spaces*, Nonlinear Anal. **74** (2011), 4387–4391.

[2] K. Aoyama, S. Iemoto, F. Kohsaka, and W. Takahashi, *Fixed point and ergodic theorems for λ-hybrid mappings in Hilbert spaces*, J. Nonlinear Convex Anal. **11** (2010), 335–343.

[3] F. E. Browder, *Convergence theorems for sequences of nonlinear operators in Banach spaces*, Math. Z. **100** (1967), 201–225.

[4] R. E. Bruck Jr., *Nonexpansive projections on subsets of Banach spaces*, Pacific J. Math. **47** (1973), 341–355.

[5] F. Kohsaka, *Existence of fixed points of nonspreading mappings with Bregman distances*, Advances in Intelligent and Soft Computing **100**, pp. 403–410, 2011.

[6] F. Kohsaka and W. Takahashi, *Fixed point theorems for a class of nonlinear mappings related to maximal monotone operators in Banach spaces*, Arch. Math. (Basel) **91** (2008), 166–177.

Sep–13 0930–1010

Matrices and tensors of smallest possible norm

Andrew Tonge

Kent State University, Kent OH 44242, USA
Email **tonge @ math.kent.edu**

In many areas of analysis, average behavior is close to extreme. This phenomenon is well-known for operator norms of matrices and tensors whose entries are independent random variables whose values are 1 and -1 with equal probability - and it has a variety of applications in functional analysis and operator theory. In this talk, we identify the least possible values for operator norms of square matrices whose entries are 1 or -1, modulo the Hadamard Conjecture. (Joint work with Lizbee Collins-Wildman and Matt Hoffman.)

Diametrically maximal sets in Banach spaces

Pier Luigi Papini

Bologna - Italy
Email **pierluigi.papini @ unibo.it**

Let X be a Banach space. A set D is said to be diametrically maximal, (DM) for short, if the addition of any point to D increases its diameter. This notion is one century old: initially it was defined as an alternative definition, for closed convex sets, to constant width, (CW) for short: a notion already studied in classical contexts from a lot of time. Indeed, (DM) and (CW) coincide in Euclidean spaces; the same (less trivial fact) is true for Hilbert spaces. For Minkowski spaces, the two definitions agree only in dimension 2 (this is known from almost half century), the class of (DM) sets being larger.

The two classes of sets can be considered also in infinite-dimensional spaces, but this was practically done only in the last few decades. It turns out that in many spaces the two classes are really different. Indeed, in general, the notion of (CW) set has a more geometrical appeal; the study of (DM) sets can involve the analytical and the geometrical structure of the space. Given a sets A in X, we say that D(A) is a completion of A if it is (DM) and its diameter is the same as the diameter of A.

Concerning (DM) sets, the following questions can be considered: Given a bounded set A in X, how can we construct a completion of A? When is such a completion unique? Can we construct a completion preserving some properties of A? Can we indicate a canonical completion and/or a nice map sending A to some completion of A? Is the class (DM) closed with respect to the Hausdorff metric?

In this talk we indicate some results and some problems concerning the above questions. Moreover, we indicate several properties for closed bounded sets which are akin to the two above notions, and we indicate the relations among them.

In recent times relevant contributions to this topic have been given, among others, by J.P. Moreno, R.R. Phelps, R. Schneider, H. Martini, M.V. Balashov, E.S. Polovinkin.

Sep–13 1110–1150

Fixed point set of measures and positive definite functions on a locally compact group

Anthony To-Ming Lau

University of Alberta
Email **anthonyt @ ualberta.ca**

In this talk I will explore some recent results on fixed point sets of positive measures and positive definite functions on a locally compact group. Some related recent works on the structure of fixed point set in the group von Neumann algebra of power bounded elements of the Fourier Stieltjes algebra of a locally compact group will also be discussed.

Sep–13 1150–1220

Schur type properties in Banach lattices–a survey

Witold Wnuk

Faculty of Mathematics and Computer Science, A. Mickieiwcz University,
Poznań, Poland
Email **wnukwit @ amu.edu.pl**

Several types of the Schur property is considered in the theory of Banach lattices. Namely, a Banach lattice $E = (E, \| \cdot \|)$ has

- the Schur property, if $x_n \to 0$ weakly, then $\|x_n\| \to 0$,
- the positive Schur property, whenever $0 \leq x_n \to 0$ weakly implies $\|x_n\| \to 0$,
- the strong Schur property if there exists a number $K > 0$ such that for all $\delta \in (0, 2]$ every δ-separated sequence in the unit ball contains a subsequence $K\delta$-equivalent to the standard ℓ^1-basis,
- the dual positive Schur property, if $0 \leq f_n \to 0$ in the weak* topology, then $\|f_n\| \to 0$.

We present various characterizations of properties mentioned above as well as examples of Banach lattices having these properties. We concentrate on structural and operator characterizations, i.e., we show which properties of a Banach lattices E imply (or are equivalent to) a respective type of the Schur property, and which relationships between suitable classes of operators on E are responsible for the Schur properties.

Sep–13 1400–1430

How to caluculate the James constants of Banach spaces?

Kichi-Suke Saito

Department of Mathematics, Faculty of Science, Niigata University,
Niigata 950-2181, Japan
Email **saito @ math.sc.niigata-u.ac.jp**

The notion of the James constant of Banach spaces was introduced by Gao and Lau and it has been studied by several authors. The James constant $J(X)$ of a Banach space X is defined by

$$J(X) = \sup\{\min\{\|x+y\|, \|x-y\|\} : x, y \in X, \|x\| = \|y\| = 1\}.$$

It is well-known that $\sqrt{2} \leq J(X) \leq 2$ for any banach space X, and $J(X) < 2$ if and only if X is uniformly non-square.

This talk is a survey of recent results about the calculations of several geometric constants of normed linear spaces. In particular, we present how to calculate the James constant of $\mathbb{R}^2$ with absolute normalized norms. Further, we remark about the properties of James constant.

This is the joint work with Naoto Komuro and Ken-Ichi Mitani.

Banach spaces which are semi-uniform Kadec–Klee

Satit Saejung* and Ji Gao

Department of Mathematics, Faculty of Science, Khon Kaen University,
Khon Kaen 40002, Thailand
Email **saejung @kku.ac.th**

Department of Mathematics, Community College of Philadelphia,
Philadelphia, PA, 19130-3991, USA
Email **jgao @ccp.edu**

Inspired by the concept of U-spaces introduced by Lau [2] we introduced the class of semi-uniform Kadec–Klee spaces, which is a uniform version of semi Kadec–Klee spaces studied by Vlasov [3]. This class of spaces is a wider subclass of spaces with weak normal structure and hence generalizes many known results in the literature. We give a characterization for a certain direct sum of Banach spaces to be semi-uniform Kadec–Klee and use this result to construct a semi-uniform Kadec–Klee space which is not uniform Kadec–Klee. At the end of the paper, we give a remark concerning the uniformly alternative convexity or smoothness introduced by Kadets et al. [1].

[1] V. M. Kadets, R. V. Shvidkoy, G. G. Sirotkin, and D. Werner, Espaces de Banach ayant la propriété de Daugavet, Comptes Rendus de l'Académie des Sciences. Série I 325(12) 1997, 1291–1294 (French).

[2] K. S. Lau, Best approximation by closed sets in Banach spaces, Journal of Approximation Theory 23 (1) 1978, 29–36.

[3] L. P. Vlasov, Some theorems on Chebyshev sets, Mathematical Notes of the Academy of Sciences of the USSR 11(2) 1972, 87–92.

Partially ℓ_1-norms and convex functions

Mikio Kato

Department of Mathematics, Faculty of Engineering, Shinshu University,
Nagano 380-8553, Japan
Email **katom @shinshu-u.ac.jp**

Every absolute normalized norm on $\mathbb{C}^N$ is constructed from a convex function ψ satisfying certain conditions on the convex set $\Delta_N = \{(s_1, \cdots, s_{N-1}) \in \mathbb{R}^{N-1} : \sum_{i=1}^{N-1} s_i \le 1,\ s_i \ge 0\}$ ([6, 1]). The class of such convex functions is dented by Ψ_N. The ψ-direct sum of N Banach spaces is their direct sum equipped with the norm $\|(x_1, \ldots, x_N)\|_\psi := \|(\|x_1\|, \ldots, \|x_N\|)\|_\psi$, where the right side $\|\cdot\|_\psi$ is the corresponding norm to $\psi \in \Psi_N$ ([4, 7]).

The starting point of this work is the following ([3]): *The ψ-direct sum $X \oplus_\psi Y$ of Banach spaces X and Y, $\psi \in \Psi_2$, is uniformly non-square (UNSQ) if and only if X and Y are UNSQ and $\psi \ne \psi_1, \psi \ne \psi_\infty$,* where ψ_1 and ψ_∞ are the corresponding convex functions to ℓ_1- and ℓ_∞-norms on $\mathbb{C}^2$, respectively. For N Banach spaces the situation is much more complicated. Dowling and Saejung [2] presented a characterization of the *UNSQ* ψ-direct sum for 3 Banach spaces from Z-direct sum approach, where they mentioned that it looks quite complicated for the case $N \ge 4$.

Our approach is to find out a subclass of Ψ_N including ψ_1, ψ_∞ which should be excluded to extend the above characterization in [3]. As a step (not small) to the goal we shall introduce a subclass $\Psi_N^{(1)}$ of convex functions which yield *ℓ_1-like norms*, or one may say *partially ℓ_1-norms* on $\mathbb{C}^N$. We shall present a sequence of their characterizations and several applications.

This is a joint work with Prof. Takayuki Tamura, Chiba Univ., Japan.

[1] F. F. Bonsall and J. Duncan, *Numerical Ranges II*, London Math. Soc. Lecture Note Ser. **10** (1973).

[2] P. N. Dowling and S. Saejung, Non-squareness and uniform non-squareness of Z-direct sums, J. Math. Anal. Appl. **369** (2010), 53–59.

[3] M. Kato, K.-S. Saito and T. Tamura, Uniform non-squareness of ψ-direct sums of Banach spaces $X \oplus_\psi Y$, Math. Inequal. Appl. **7** (2004), 429–437.

[4] M. Kato, K.-S. Saito and T. Tamura, On the ψ-direct sums of Banach spaces and convexity, J. Aust. Math. Soc. **75** (2003), 413–422.

[5] M. Kato and T. Tamura, Weak nearly uniform smoothness of ψ-direct sums $(X_1 \oplus \cdots \oplus X_N)_\psi$, submitted.

[6] K.-S. Saito, M. Kato and Y. Takahashi, On absolute norms on $\mathbb{C}^n$, J. Math. Anal. Appl. **252** (2000), 879–905.

[7] Y. Takahashi, M. Kato and K.-S. Saito, Strict convexity of absolute norms on $\mathbb{C}^2$ and direct sums of Banach spaces, J. Inequal. Appl. **7** (2002), 179–186.

Skewness and some geometrical constants of Banach spaces

Ken-Ichi Mitani

Department of Systems Engineering, Okayama Prefectural University,
Soja 719-1197, Japan
Email **mitani @cse.oka-pu.ac.jp**

Let X be a real Banach space. In this talk we present recent results on the skewness of X, especially in connection with the modulus of smoothness and the James constant. This is a joint work with K.-S. Saito and Y. Takahashi.

A structure of n-dimensional normed linear spaces

Ryotaro Tanaka* and Kichi-Suke Saito

Department of Mathematical Science,
Graduate School of Science and Technology,
Niigata University, Niigata 950-2181, Japan
Email **ryotarotanaka @ m.sc.niigata-u.ac.jp**

Department of Mathematics, Faculty of Science,
Niigata University, Niigata 950-2181 Japan
Email **saito @ math.sc.niigata-u.ac.jp**

There are many norms on the n-dimensional vector space $\mathbb{R}^n$. A norm $\|\cdot\|$ on $\mathbb{R}^n$ is said to be absolute if $\|(x_1, x_2, \ldots, x_n)\| = \|(|x_1|, |x_2|, \ldots, |x_n|)\|$ for all $(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$, and normal if $\|\cdot\|_\infty \leq \|\cdot\| \leq \|\cdot\|_1$. Recently, Alonso showed that every two-dimensional normed space is isometrically isomorphic to a generalized Day-James space which was introduced by Nilsrakoo and Saejung. In fact, a normed space $(\mathbb{R}^2, \|\cdot\|)$ is a generalized Day-James space if and only if the norm $\|\cdot\|$ is normal.

In this talk, we consider the above result in case of n-dimensional normed linear spaces.

The orthogonal decomposition of Banach spaces and nonlinear analytic approach to the splitting theorem of Jacobs-deLeeuw-Glicksberg

Takashi Honda

Department of Mathematics, Faculty of Education / Graduate School of Education, Iwate University,
3-18-33 Ueda, Morioka, Iwate, 020-8550, Japan
Email **thonda7 @iwate-u.ac.jp**

In this talk, we introduce an orthogonal decomposition in a Banach space which is an extension of the orthogonal complemented subspace decomposition of a Hilbert space. By using it, we can define an orthogonal projection of a Banach space as a nonlinear retraction. We show that all contractive linear projections of a Banach space are orthogonal projections. Recently, we study nonlinear analytic methods for linear contractive semigroups in Banach spaces and apply them to the splitting theorem of Jacobs-de Leeuw-Glicksberg. Using these results, we obtain the extension of Lin's proposition for a group of linear operators to a semigroup.

[1] T. Honda and W. Takahashi, *Norm One Projections and Generalized Conditional Expectations*, Sci. Math. Jpn. **69** (2009), 303–313.

[2] T. Honda, W. Takahashi and J. C. Yao, *Nonexpansive retractions onto closed convex cones in Banach spaces*, Taiwanese J. Math., **14** (2010), no. 3B, 1023–1046.

[3] T. Honda, *Nonlinear analytic approach to the splitting theorem of Jacobs-deLeeuw-Glicksberg*, Nonlinear Anal., to appear.

B_σ-function space estimates for some operators

Katsuo Matsuoka

College of Economics, Nihon University,
Misaki-cho, Chiyoda-ku, Tokyo 101-8360, Japan
Email **katsu.m @nihon-u.ac.jp**

For $\sigma \in [0,\infty)$, $p \in [1,\infty)$ and $\lambda \in \mathbb{R}$, we define B_σ-Morrey space, i.e., $B_\sigma(L_{p,\lambda})(\mathbb{R}^n)$ as the sets of all functions f on $\mathbb{R}^n$ such that

$$\|f\|_{B_\sigma(L_{p,\lambda})} = \sup_{r\geq 1} \frac{1}{r^\sigma} \left\{ \sup_{Q(x,s)\subset Q_r} \frac{1}{s^\lambda} \left(\frac{1}{|Q(x,s)|} \int_{Q(x,s)} |f(y)|^p dy \right)^{1/p} \right\} < \infty,$$

where $Q_r = \{y = (y_1, y_2, \cdots, y_n) \in \mathbb{R}^n : \max_{1\leq i\leq n} |y_i| < r\}$ or $Q_r = \{y \in \mathbb{R}^n : |y| < r\}$, and $Q(x,s) = \{x + y : y \in Q_s\}$. And then we consider the boundedness of some operators on the $B_\sigma(L_{p,\lambda})(\mathbb{R}^n)$, which unify a series of results on the boundedness on several function spaces.

The Dunkl-Williams constant of some Banach spaces

Hiroyasu Mizuguchi*, Kichi-Suke Saito and Ryotaro Tanaka

Department of Mathematical Sciences,
Graduate School of Science and Technology, Niigata University,
Niigata, 950-2181 Japan
Email **mizuguchi @m.sc.niigata-u.ac.jp**

Department of Mathematics, Faculty of Science, Niigata University,
Niigata 950-2181, Japan
Email **saito @math.sc.niigata-u.ac.jp**

Department of Mathematical Sciences,
Graduate School of Science and Technology, Niigata University,
Niigata, 950-2181 Japan
Email **ryotarotanaka @m.sc.niigata-u.ac.jp**

In 1964, C. F. Dunkl and K. S. Williams showed that for any nonzero elements x, y in a Banach space X,

$$\left\| \frac{x}{\|x\|} - \frac{y}{\|y\|} \right\| \leq \frac{4\|x-y\|}{\|x\|+\|y\|}.$$

This inequality is called the Dunkl-Williams inequality and have been studied.

In 2008, A. Jimenez-Melado et. al defined the Dunkl-Williams constant $DW(X)$ of a Banach space X:

$$DW(X) = \sup \left\{ \frac{\|x\|+\|y\|}{\|x-y\|} \left\| \frac{x}{\|x\|} - \frac{y}{\|y\|} \right\| \;\middle|\; x, y \in X,\ x, y \neq 0,\ x \neq y \right\}.$$

We summarize some basic properties of the Dunkl-Williams constant.
(i) For any Banach space X, $2 \leq DW(X) \leq 4$.
(ii) $DW(X) = 2$ if and only if X is an inner product space.
(iii) $DW(X) < 4$ if and only if X is uniformly non-square, that is, there exits $\delta > 0$ such that $\|x+y\| > 2(1-\delta)$ and $\|x\| = \|y\| = 1$ implies $\|x-y\| < 2(1-\delta)$.

In this talk, we present how to calculate this constant. As an application, we calculate the Dunkl-Williams constant of some Banach spaces.

Convergence of iterative sequences for nonlinear mappings

Sachiko Atsushiba

Department of Science Education, Graduate School of Education Science of Teaching and Learning, University of Yamanashi, 4-4-37, Takeda Kofu, Yamanashi 400-8510, Japan
Email **asachiko @yamanashi.ac.jp**

In this talk, we deal with convergence theorems for families of nonlinear mappings in Hilbert spaces or Banach spaces. We first prove nonlinear mean convergence theorems for nonlinear mappings (see [1–3]). We also prove convergence theorems for nonlinear mappings by the Mann iteration process.

[1] J.B. Baillon, *Un theoreme de type ergodique pour les contractions non lineaires dans un espace de Hilbert*, C. R. Acad. Sci. Paris Sger., A-B **280** (1975), 1511–1514.

[2] P. Kocourek, W. Takahashi, and J.-C. Yao, *Fixed point theorems and ergodic theorems for nonlinear mappings in Banach spaces*, Adv. Math. Econ. **15** (2011), 67–88.

[3] W. Takahashi and Y. Takeuchi, *Nonlinear ergodic theorem without convexity for generalized hybrid mappings in a Hilbert space*, J. Nonlinear Convex Anal. **12** (2011), 399–406.

The duality of a generalized Bergman space

Kamthorn Chailuek* and Marisa Senmoh

Department of Mathematics and Statistics, Prince of Songkla University,
Hatyai 90112 , Thailand
Email **kamthorn.c @psu.ac.th**

Department of Mathematics and Statistics, Prince of Songkla University,
Hatyai 90112 , Thailand
Email **5210220064 @email.psu.ac.th**

We study the Bergman space $HL^2(B,\, dv_\alpha)$ consisting of all holomorphic function f on the unit ball $B = \{z\colon |z| < 1\}$ which f is square-integrable with respect to measure $dv_\alpha = \dfrac{\Gamma(\alpha+2)}{\pi\Gamma(\alpha+1)}(1-|z|^2)^\alpha$. The spaces are non-zero if and only if $\alpha > -1$. However the formula for reproducing kernel is given by $K_\alpha(z,w) = \dfrac{1}{\pi(1-\langle z,w\rangle)^{\alpha+2}}$ which is positive definite when $\alpha > -2$. This indicates that we can define spaces with the same formula for the reproducing kernel when $\alpha > -2$. By using a Sobolev-type norm, a generalized Bergman space $H(B,\alpha) := \{f \in HL^2(B,\, dv_{\alpha+2}) : z\frac{df}{dz} \in HL^2(B,\, dv_{\alpha+2})\}$ is the same as $HL^2(B,\, dv_\alpha)$ when $\alpha > -1$ but still non-zero when $-2 < \alpha \leq -1$.

In this talk, we will present the dual space of $H(B,\alpha)$ which can be identified with a Bergman space $HL^2(B,\, dv_\beta)$ for some β.

[1] Chailuek, K. and Hall, B., Toeplitz operators on generalized Bergman spaces, Integral Equations and Operator Theory, 66(2010), 53 − 77.

[2] Senmoh, M., Another aspect of a generalized Bergman space, Master Thesis, Graduate School, Prince of Songkla University, 2012.

[3] Zhu K., Spaces of holomorphic functions in the unit ball, Springer, 2010.

Structure of Cesàro function spaces

Lech Maligranda

Department of Engineering Sciences and Mathematics
Luleå University of Technology
971 87 Luleå, Sweden
Email **lech.maligranda @ltu.se**

The *Cesàro function spaces* $Ces_p(I)$ on both $I = [0,1]$ and $I = [0,\infty)$ are the classes of Lebesgue measurable real functions f on I such that

$$\|f\|_{C(p)} = \left[\int_I \left(\frac{1}{x}\int_0^x |f(t)|\,dt\right)^p dx\right]^{1/p} < \infty \quad \text{for} \quad 1 \le p < \infty,$$

and

$$\|f\|_{C(\infty)} = \sup_{x\in I,\ x>0} \frac{1}{x}\int_0^x |f(t)|\,dt < \infty \quad \text{for} \quad p = \infty.$$

For $1 < p < \infty$ spaces $Ces_p(I)$ are separable, strictly convex and not symmetric. They, in the contrast to the sequence spaces, are not reflexive and do not have the fixed point property (cf. [1]). The *Cesàro sequence spaces* ces_p for $1 < p < \infty$ are defined as the set of all real sequences $x = \{x_k\}$ such that

$$\|x\|_{c(p)} = \left[\sum_{n=1}^{\infty}\left(\frac{1}{n}\sum_{k=1}^{n}|x_k|\right)^p\right]^{1/p} < \infty \quad \text{for} \quad 1 \le p < \infty,$$

and

$$\|x\|_{c(\infty)} = \sup_{n\in\mathbb{N}}\left(\frac{1}{n}\sum_{k=1}^{n}|x_k|\right) < \infty \quad \text{for} \quad p = \infty.$$

They are separable, reflexive, not symmetric and not B-convex Banach spaces, but they have the fixed point property.

The structure of the Cesàro function spaces $Ces_p(I)$ is investigated. Their dual spaces, which equivalent norms have different description on $[0,1]$ and $[0,\infty)$, are described. The spaces $Ces_p(I)$ for $1 < p < \infty$ are not isomorphic to any $L^q(I)$ space with $1 \le q \le \infty$. They have "near zero" complemented subspaces isomorphic to l^p and "in the middle" contain an asymptotically isometric copy of l^1 and also a copy of $L^1[0,1]$. They do not have Dunford-Pettis property. Cesàro function spaces on $[0,1]$ and $[0,\infty)$ are isomorphic for $1 < p < \infty$. Moreover, the Rademacher functions span in $Ces_p[0,1]$ for $1 \le p < \infty$ a space which is isomorphic to l^2 since the equivalence

$$\|\sum_{k=1}^{n} a_k\, r_k\|_{C(p)} \approx \left(\sum_{k=1}^{n} a_k^2\right)^{1/2}$$

is true for any real numbers $a_1, a_2, ..., a_n$ and any $n \in \mathbb{N}$. This subspace is uncomplemented in $Ces_p[0,1]$. In the space $Ces_\infty[0,1]$, and also in its p-convexification $Ces_\infty^{(p)}[0,1] := K_p$ with $1 \le p < \infty$, the corresponding equivalence is

$$\| \sum_{k=1}^{n} a_k r_k \|_{K_p} \approx \left(\sum_{k=1}^{n} a_k^2 \right)^{1/2} + \max_{1 \le m \le n} | \sum_{k=1}^{m} a_k |.$$

The talk is based on joint papers with Sergey V. Astashkin.

[1] S. V. Astashkin and L. Maligranda, *Cesàro function spaces fail the fixed point property*, Proc. Amer. Math. Soc. **136** (2008), no. 12, 4289–4294.

[2] S. V. Astashkin and L. Maligranda, *Structure of Cesàro function spaces*, Indag. Math. **20** (2009), no. 3, 329–379.

[3] S. V. Astashkin and L. Maligranda, *Rademacher functions in Cesàro type spaces*, Studia Math. **198** (2010), no. 3, 235–247.

[4] S. V. Astashkin and L. Maligranda, *Geometry of Cesàro function spaces*, Funktsional. Anal. i Prilozhen. **45** (2011), no. 1, 79–83.

[5] S. V. Astashkin, *On the geometric properties of Cesàro spaces*, Mat. Sb. **203** (2012), no. 4, 61–80; English transl. in Sb. Math. **203** (2012), no. 4, 514–533.

[6] G. Bennett, *Factorizing the Classical Inequalities*, Mem. Amer. Math. Soc. **120**, Providence 1996.

[7] A. Kamińska and T. Kubiak, *On the dual of Cesàro function space*, Nonlinear Analysis **75** (2012), no. 5, 2760–2773.

[8] A. Kufner, L. Maligranda and L.-E. Persson, *The Hardy Inequality. About its History and Some Related Results*, Vydavatelski Servis Publishing House, Pilzen 2007.

[9] L. Maligranda, N. Petrot and S. Suantai, *On the James constant and B-convexity of Cesàro and Cesàro-Orlicz sequence spaces*, J. Math. Anal. Appl. **326** (2007), no. 1, 312–331.

The Cesàro operator on Hardy spaces: extensions and optimal domains

Guillermo P. Curbera

Facultad de Matemáticas,
Universidad de Sevilla,
Aptdo 1160,
41080 Sevilla,
Spain
Email **curbera @ us.es**

The classical Cesàro operator, given by

$$\mathcal{C}(f)(z) := \sum_{n=0}^{\infty} \Big(\frac{1}{n+1} \sum_{k=0}^{n} a_k \Big) z^n,$$

with $f(z) = \sum_0^\infty a_k z^k$ an analytic function on the open unit disc D, is bounded on the Hardy space H^p, for every $1 \leq p < \infty$. For each $1 \leq p < \infty$, there exist analytic functions $f \notin H^p(D)$ with $\mathcal{C}(f) \in H^p(D)$. We discuss the identification and properties of the (Banach) space of analytic functions $[\mathcal{C}, H^p]$ consisting of *all* analytic functions that $\mathcal{C}$ maps into $H^p(D)$.

It is shown that $[\mathcal{C}, H^p]$ contains classical Banach spaces of analytic functions X, genuinely larger that H^p, such that $\mathcal{C}$ has a continuous H^p-valued extension to X. An important feature of $[\mathcal{C}, H^p]$ is that functions $f \in [\mathcal{C}, H^p]$ have a *growth characterization*. Namely, for $1 < p < \infty$,

$$f \in [\mathcal{C}, H^p] \iff \int_0^{2\pi} \Big(\int_0^1 \frac{|f(re^{i\theta})|^2}{|1 - re^{i\theta}|^2} (1-r)\, dr \Big)^{p/2} d\theta < \infty.$$

Particular attention is given to the subspace of $[\mathcal{C}, H^2]$ consisting on all functions which are the *unconditional sum* of their Taylor series.

The work presented is joint with Werner J. Ricker from the Katholische Universitt Eichstätt-Ingolstadt (Germany).

[1] G.P. Curbera and W.J. Ricker, Extensions of the classical Cesàro operator on Hardy spaces, Math. Scand., 108 (2011) 637-653.

[2] ———, Solid extension of the Cesàro operator on the Hardy space $H^2(D)$, preprint.

Some exotic constructions in Banach spaces

Kazimierz Goebel

Maria Curie Sklodowska University,
Institute of Mathematics, 20-031 Lublin, Poland
Email **goebel @ hektor.umcs.lublin.pl**

It is obvious and well known fact that the geometry and topology of finite dimensional Banach spaces changes substantially when we pass to the case of infinite dimension. This is caused mainly by the fact that bounded and closed subsets of infinite dimensional space are not necessarily compact.

Especially all balls are not compact. In consequence of that, many classical theorems valid in finite dimensional spaces fail in this more general setting. Within the category of infinite dimensional Banach spaces, there are also diferences caused by the regularity of geometries induced by the selection of the norm.

We present here some examples and open problems.

Some inclusion relations of Orlicz-Morrey spaces and the Hardy-Littlewood maximal function

Hiro-o Kita

Department of Mathematics, Kagoshima University,
20-6 Korimoto 1-chome, Kagoshima 890-0065, Japan
Email **hkita @ edu.kagoshima-u.ac.jp**

We introduce a concept of generalized Orlicz-Morrey spaces and state a necessary and sufficient condition related to inclusion relations of these spaces. And a necessary and sufficient condition of the boundedness of the Hardy-Littlewood maximal operator in generalized Orlicz-Morrey spaces is given. This result is a generalization of Theorem 5.1 in [4]

Theorem 1 *Let Φ be a Young function and $\phi, \psi \in \mathcal{G}_1$. And let Ψ be a p-convex function for some $0 < p \leq 1$. Then (i) and (ii) are equivalent:*

(i) The inclusion relation $L^{(\Phi,\phi)}(\mathbb{R}^n) \subseteq L^{(\Psi,\psi,p)}(\mathbb{R}^n)$ holds.

(ii) The following properties (p-1) and (p-2) hold.

(p-1) There exist constants $C_1 > 1$ and $C_2 > 1$ such that

$$\Psi(t) \leq C_1 \psi(\phi^{-1}(\Phi(C_2 t))) \quad \textit{for all} \quad 0 < t < \infty\,. \tag{1}$$

(p-2) There exist constants $A_1 > 1$ and $A_2 > 1$ such that

$$\Psi\left(\frac{s}{A_1}\right) \leq A_2 \Phi(s) \cdot \frac{\psi(r)}{\phi(r)} \tag{2}$$

for all r, s satisfying $\Phi^{-1}(\phi(r)) < s < \sup_{0<u<\infty} \Phi^{-1}(\phi(u))$.

[1] L. Maligranda, *Orlicz spaces and interpolation*, Sem. Math. 5, Depto de Mat. Univ. Estadual de Campinas, Brasil, 1989.

[2] C. B. Morrey, *On the solutions of quasi-linear elliptic partial differential equations*, Trans. Amer. Math. Soc. **43** (1938), 126-166.

[3] E. Nakai, *Generalized fractional integrals on Orlicz–Morrey spaces*, in: Banach and Function Spaces (Kitakyushu, 2003), Yokohama Publ., Yokohama 2004, 323–333.

[4] ———, *Orlicz-Morrey spaces and the Hardy -Littlewood maximal function*, Studia Math. **188** (2008), 193–221.

[5] W. Orlicz, *Über eine gewisse Klasse von Räumen vom Typus B*, Bull. int'l. de l'Acad. Pol. série A **8** (1932), 207–220.

Interpolation using the unit square and limiting real methods

Luz M. Fernández-Cabrera

Departamento de Matemática Aplicada
Escuela de Estadística, Universidad Complutense de Madrid,
28040 Madrid. Spain.
Email **luz_fernandez-c @ mat.ucm.es**

We deal with the interpolation methods $\bar{\mathbb{A}}_{(\alpha,\beta),q;J}$, $\bar{\mathbb{A}}_{(\alpha,\beta),q;J}$ associated to the unit square (see [6, 5]). Let A_0 and A_1 be Banach spaces with $A_0 \hookrightarrow A_1$ and let $\bar{\mathbb{A}} = (A_0, A_1, A_1, A_0)$ be the 4-tuple obtained by sitting A_0 on $(0,0)$ and $(1,1)$, and A_1 on $(1,0)$ and $(0,1)$. It is shown in [2] and [1] that interpolating $\bar{\mathbb{A}}$ by the methods defined by the square with (α, β) in the diagonals, then the resulting spaces are real interpolation spaces $(A_0, A_1)_{\theta,q}$ where $0 < \theta < 1$, $1 \leq q \leq 1$ and limiting real spaces where $\theta = 0, 1$. In this talk, we consider the case of a general Banach couple $\bar{A} = (A_0, A_1)$, not necessary ordered. For this aim, we first introduce limiting real methods that work for arbitrary Banach couples. Then we show that the spaces generated by the K-method (respectively, J-method) defined by the unit square and $\bar{\mathbb{A}}$ turn out to be sums (respectively, intersections) of limiting spaces and real interpolation spaces.

Results are part of joint works with F. Cobos and P. Silvestre [3, 4].

[1] F. Cobos, L.M. Fernández-Cabrera, T. Kühn and T. Ullrich, On an extreme class of real interpolation spaces, J. Funct. Anal. **256** (2009) 2321-2366.

[2] F. Cobos, L.M. Fernández-Cabrera and J. Martín, Some reiteration results for interpolation methods defined by means of polygons, Proc. Royal Soc. Edinb. **138A** (2008) 1179-1195.

[3] F. Cobos, L.M. Fernández-Cabrera and P. Silvestre, New limiting real interpolation methods and their connection with the methods associated to the unit square, Math. Nachr. (to appear).

[4] F. Cobos, L.M. Fernández-Cabrera and P. Silvestre, Limiting J-spaces for general couples, Z. Anal. Anwendungen (to appear).

[5] F. Cobos and J. Peetre, Interpolation of compact operators: The multidimensional case, Proc. London Math. Soc. **63** (1991) 371-400.

[6] D.L. Fernandez, Interpolation of 2^n Banach spaces, Studia Math. **45** (1979) 175-201.

Hardy spaces with variable exponents

Yoshihiro Sawano

Department of Mathematics and informations, Tokyo Metropolitan University
Email **ysawano @ tmu.ac.jp**

In this talk the speaker would like to convey the notion of Hardy spaces are important in analysis. The speaker will explain the motivation and the definition of Hardy spaces. One of the advantages is the atomic decomposition. As an evidence, the speaker will propose an application.

To realize this purpose, we use the variable Lebesgue spaces, which are believed to be most difficult function spaces in harmonic analysis.

Let $L^{p(\cdot)}(\mathbf{R}^n)$ be the set of all measurable functions f for which the quasi-norm

$$\|f\|_{L^{p(\cdot)}} = \inf\left\{\lambda > 0 \, : \, \int_{\mathbf{R}^n} \left(\frac{|f(x)|}{\lambda}\right)^{p(x)} dx \leq 1\right\}$$

is finite. One defines variable Hardy spaces by using the following norm

$$\|f\|_{L^{p(\cdot)}} \equiv \left\| \sup_{t>0} |e^{t\Delta} f| \right\|_{L^{p(\cdot)}}.$$

We aim here to prove the following fact.

Theorem 1

1. *The variable Hardy spaces admits the maximal characterization.*
2. *The variable Hardy spaces admits the atomic characterization.*
3. *Singular integral operators are bounded.*
4. *The variable Hardy spaces admits the Littlewood-Paley characterization.*
5. *If* $\sup p(x) \leq 1$*, then the dual can be characterized.*

As an application of the atomic decomposition, the speaker will mention how we apply it to the boundedness of operators. We try to obtain the Olsen inequality, which is of the form

$$\|f \cdot g\|_X \leq \|g\|_Y \|(-\Delta)^{\alpha/2} f\|_Z$$

for some Banach spaces.

This work is partly based upon [1] and some related works in preparation.

[1] Eiichi Nakai and Yoshihiro Sawano, Hardy spaces with variable exponents and generalized Campanato spaces, Journal of Functional Analysis 262 (2012) 3665–3748.

Generalized Morrey spaces and generalized fractional integrals

Eiichi Nakai

Department of Mathematics, Ibaraki University
Mito, Ibaraki 310-8512, Japan
Email **enakai @ mx.ibaraki.ac.jp**

For $x = (x_1, x_2, \cdots, x_n) \in \mathbb{R}^n$ and $r > 0$, let

$$Q(x,r) = \left\{ y = (y_1, y_2, \cdots, y_n) \in \mathbb{R}^n : \max_{1 \le i \le n} |y_i - x_i| < r \right\}.$$

For $p \in [1, \infty)$ and $\phi : (0, \infty) \to (0, \infty)$, let $L_{p,\phi}(\mathbb{R}^n)$ be the set of all functions f such that the following functional is finite:

$$\|f\|_{L_{p,\phi}(\mathbb{R}^n)} = \sup_{Q(x,r)} \frac{1}{\phi(r)} \left(\frac{1}{|Q(x,r)|} \int_{Q(x,r)} |f(y)|^p \, dy \right)^{1/p}.$$

Assume that $\lim_{r \to 0} \phi(r) = \infty$, $\lim_{r \to \infty} \phi(r) = 0$ and

$$\phi(r) \le C\phi(s), \quad s^{n/p}\phi(s) \le C r^{n/p}\phi(r) \quad \text{for } 0 < s < r.$$

For $\rho : (0, \infty) \to (0, \infty)$ with appropriate conditions, let

$$I_\rho f(x) = \int_{\mathbb{R}^n} \frac{\rho(|x-y|)}{|x-y|} f(y) \, dy.$$

Gunawan [2] proved that the operator I_ρ is bounded from $L_{p,\phi}(\mathbb{R}^n)$ to $L_{q,\phi^{p/q}}(\mathbb{R}^n)$, if $p < q$ and

$$\phi(r) \int_0^r \frac{\rho(t)}{t} \, dt + \int_r^\infty \frac{\phi(t)\rho(t)}{t} \, dt \le C\phi(r)^{p/q} \quad \text{for} \quad r > 0.$$

This is a generalization of Adams' result in [1]. In this talk we extend Gunawan's result to generalized Morrey spaces with variable growth condition. This is also an extension of the result in [3].

[1] D. R. Adams, *A note on Riesz potentials*, Duke Math. J. 42 (1975), no. 4, 765–778.

[2] H. Gunawan, *A note on the generalized fractional integral operators*, J. Indonesian Math. Soc. (MIHMI) **9**(1) (2003), 39–43.

[3] E. Nakai, *Hardy-Littlewood maximal operator, singular integral operators and the Riesz potentials on generalized Morrey spaces*, Math. Nachr. 166 (1994), 95–103.

Sep–14 1550–1610

Generalized metric adjusted skew information and uncertainty relation

Kenjiro Yanagi

Graduate School of Science and Engineering, Yamaguchi University,
Ube 755-8611, Japan
Email **yanagi @yamaguchi-u.ac.jp**

Let $M_n(\mathbb{C})$(resp. $M_{n,sa}(\mathbb{C})$) be the set of all $n \times n$ complex matrices (resp. all $n \times n$ self-adjoint matrices), endowed with the Hilbert-Schmidt scalar product $\langle A, B\rangle = Tr(A^*B)$. Let $M_{n,+}(\mathbb{C})$ be the set of strictly positive elements of $M_n(\mathbb{C})$ and $M_{n,+,1}(\mathbb{C})$ be the set of stricly positive density matrices, that is $M_{n,+,1}(\mathbb{C}) = \{\rho \in M_n(\mathbb{C}) | Tr\rho = 1, \rho > 0\}$. If it is not otherwise specified, from now on we shall treat the case of faithful states, that is $\rho > 0$.

A function $f : (0, +\infty) \to \mathbb{R}$ is said operator monotone if, for any $n \in \mathbb{N}$, and $A, B \in M_n$ such that $0 \leq A \leq B$, the inequalities $0 \leq f(A) \leq f(B)$ hold. An operator monotone function is said symmetric if $f(x) = xf(x^{-1})$ and normalized if $f(1) = 1$.

Definition 1 *$\mathcal{F}_{op}$ is the class of functions $f : (0, +\infty) \to (0, +\infty)$ such that*

(1) $f(1) = 1$,

(2) $tf(t^{-1}) = f(t)$,

(3) f *is operator monotone.*

Example 2 *Examples of elements of $\mathcal{F}_{op}$ are given by the following list*

$$f_{RLD}(x) = \frac{2x}{x+1}, \quad f_{WY}(x) = \left(\frac{\sqrt{x}+1}{2}\right)^2, \quad f_{BKM}(x) = \frac{x-1}{\log x},$$

$$f_{SLD}(x) = \frac{x+1}{2}, \quad f_{WYD}(x) = \alpha(1-\alpha)\frac{(x-1)^2}{(x^\alpha - 1)(x^{1-\alpha} - 1)}, \quad \alpha \in (0,1).$$

Remark *Any $f \in \mathcal{F}_{op}$ satisfies*

$$\frac{2x}{x+1} \leq f(x) \leq \frac{x+1}{2}, \quad x > 0.$$

For $f \in \mathcal{F}_{op}$ define $f(0) = \lim_{x \to 0} f(x)$. We introduce the sets of regular and non-regular functions

$$\mathcal{F}_{op}^r = \{f \in \mathcal{F}_{op} | f(0) \neq 0\}, \quad \mathcal{F}_{op}^n \{f \in \mathcal{F}_{op} | f(0) = 0\}$$

and notice that trivially $\mathcal{F}_{op} = \mathcal{F}_{op}^r \cup \mathcal{F}_{op}^n$.

Definition 3 *Let $g, f \in \mathcal{F}_{op}^r$ satisfy*

$$g(x) \geq c\frac{(x-1)^2}{f(x)}$$

for some $c > 0$. We define

$$\Delta_g^f(x) = g(x) - c\frac{(x-1)^2}{f(x)} \in \mathcal{F}_{op}^r$$

In Kubo-Ando theory of matrix means one associates a mean to each operator monotone function $f \in \mathcal{F}_{op}$ by the formula

$$m_f(A, B) = A^{1/2} f(A^{-1/2} B A^{-1/2}) A^{1/2},$$

where $A, B \in M_{n,sa}(\mathbb{C})$. Using the notion of matrix means one may define the class of monotone metrics (also said quantum Fisher informtions) by the following formula

$$\langle A, B \rangle_{\rho,f} = Tr(A \cdot m_f(L_\rho, R_\rho)^{-1}(B)),$$

where $L_\rho(A) = \rho A, R_\rho(A) = A\rho$. In this case one has to think of A, B as tangent vectors to the manifold $M_{n,+,1}(\mathbb{C})$ at the point ρ (see [9], [3]).

Definition 4 *For $A, B \in M_{n,sa}$ and $\rho \in M_{n,+,1}(\mathbb{C})$, we define the following quantities:*

$$Corr_\rho^{(g,f)}(A, B) = c\langle i[\rho, A], i[\rho, B]\rangle_{\rho,f}, \quad I_\rho^{(g,f)}(A) = Corr_\rho^{(g,f)}(A, A),$$

$$C_\rho^f(A, B) = Tr[A m_f(L_\rho, R_\rho)B], \quad C_\rho^f(A) = C_\rho^f(A, A),$$

$$U_\rho^{(g,f)}(A) = \sqrt{(C_\rho^g(A) + C_\rho^{\Delta_g^f}(A))(C_\rho^g(A) - C_\rho^{\Delta_g^f}(A))},$$

The quantity $I_\rho^{(g,f)}(A)$ and $Corr_\rho^{(g,f)}(A, B)$ are said generalized metric adjusted skew information and generalized metric adjusted correlation measure, respectively.

Then we have the following proposition.

Proposition 5 *For $A, B \in M_{n,sa}(\mathbb{C})$ and $\rho \in M_{n,+,1}(\mathbb{C})$, we have the following relations, where we put $A_0 = A - Tr[\rho A]I$ and $B_0 = B - Tr[\rho B]$.*

(1) $I_\rho^{(g,f)}(A) = I_\rho^{(g,f)}(A_0) = C_\rho^g(A_0) - C_\rho^{\Delta_g^f}(A_0)$,

(2) $J_\rho^{(g,f)}(A) = C_\rho^g(A_0) + C_\rho^{\Delta_g^f}(A_0)$,

(3) $U_\rho^{(g,f)}(A) = \sqrt{I_\rho^{(g,,f)}(A) \cdot J_\rho^{(g,f)}(A)}$.

(4) $Corr_\rho^{(g,f)}(A, B) = Corr_\rho^{(g,f)}(A_0, B_0)$.

Theorem 6 *For $f \in \mathcal{F}_{op}^r$, if*

$$g(x) + \Delta_g^f(x) \geq df(x)$$

for some $d > 0$, then it holds

$$U_\rho^{(g,f)}(A) \cdot U_\rho^{(g,f)}(B) \geq cd|Tr(\rho[A,B])|^2,$$

$$U_\rho^{(g,f)}(A) \cdot U_\rho^{(g,f)}(B) \geq 4cd|Corr_\rho^f(A,B)|^2,$$

where $A, B \in M_{n,sa}(\mathbb{C})$ and $\rho \in M_{n,+,1}(\mathbb{C})$.

Example 7 *(1) When*

$$g(x) = \frac{x+1}{2}, \quad f(x) = \alpha(1-\alpha)\frac{(x-1)^2}{(x^\alpha - 1)(x^{1-\alpha} - 1)}, \quad c = \frac{f(0)}{2}, \quad d = 2,$$

we have the results given by [13, 1].

(2) Let

$$g(x) = \left(\frac{\sqrt{x}+1}{2}\right)^2, \quad f(x) = \alpha(1-\alpha)\frac{(x-1)^2}{(x^\alpha - 1)(x^{1-\alpha} - 1)}.$$

If we assume $c = f(0)/4$ and $d = 2$, then it is probably shown by Mathematica that

$$g(x) + \Delta_g^f(x) \geq 2f(x)$$

holds for $0 < \alpha < 0.1$ or $0.9 < \alpha < 1$. And also if we assume $c = f(0)/8$ and $d = 3/2$, then it is probably shown by Mathematica that

$$g(x) + \Delta_g^f(x) \geq \frac{3}{2}f(x)$$

holds for $0 < \alpha < 1$.

(3) When

$$g(x) = \left(\frac{x^\gamma + 1}{2}\right)^{1/\gamma} \quad (\frac{3}{4} \leq \gamma \leq 1), \quad f(x) = \left(\frac{\sqrt{x}+1}{2}\right)^2,$$

$$c = \frac{f(0)}{4}, \quad d = 2,$$

it is shown that $g(x) + \Delta_g^f(x) \geq 2f(x)$ holds by Mathematica.

[1] Furuichi, S., Yanagi, K., Schrödinger uncertainty relation, Wigner-Yanase-Dyson skew information and metric adjusted correlation measure, J. Math. Anal. Appl., 388, 1147-1156(2012)

[2] Gibilisco, P., Imparato, D., Isola, T., Uncertainty principle and quantum Fisher information, II, J. Math. Phys., 48, 072109 (2007)

[3] Gibilisco, P., Hansen, F., Isola, T., On a correspondence between regular and non-regular operator monotone functions, Linear Algebra and its Applications, 430, 2225-2232 (2009)

[4] Gibilisco, P., Hiai, F., Petz, D., Quantum covariance, quantum Fisher information, and the uncertainty relations, IEEE Trans. Information Theory, 55, 439-443(2009)

[5] Gibilisco, P., Isola, T., On a refinement of Heisenberg uncertainty relation by means of quantum Fisher information, J. Math. Anal. Appl., 375, 270-275 (2011)

[6] Hansen, F., Metric adjusted skew information, Proc. Nat Acad. Sci., 105, 9909-9916 (2008)

[7] Kubo, F., Ando, T., Means of positive linear operators, Math. Ann., 246, 205-224 (1980)

[8] Luo, S., Heisenberg uncertainty relation for mixed states, Phys. Rev., A 72, 042110 (2005)

[9] Petz, D., Monotone metrics on matrix spaces, Linear Algebra and its Applications, 244, 81-96 (1996)

[10] Petz, D., Hasegawa, H., On the Riemannian metric of α-entropies of density matrices, Lett. Math. Phys., 38, 221-225 (1996)

[11] Wigner, E. P., Yanase, M. M., Information content of distribution, Proc. Nat. Acad. Sci., 49, 910-918 (1963)

[12] Yanagi, K., Uncertainty relation on Wigner-Yanase-Dyson skew information, J. Math. Anal. Appl., 365, 12-18 (2010)

[13] Yanagi, K., Metric adjusted skew information and uncertainty relation, J. Math. Anal. Appl., 380, 888-892(2011).

Applications of Fixed Point Theorems to the BCS Gap Equation for Superconductivity

Shuji Watanabe

Division of Mathematical Sciences
Graduate School of Engineering, Gunma University
4-2 Aramaki-machi, Maebashi 371-8510, Japan
Email **shuwatanabe @ gunma-u.ac.jp**

The existence and uniqueness of the solution to the BCS gap equation for superconductivity were established for each fixed temperature in the literature. But the temperature dependence of the solution was not covered. For example, how the solution varies with the temperature was not pointed out, and it was not shown that the solution is continuous with respect to the temperature.

Studying the temperature dependence of the solution to the BCS gap equation for superconductivity is very important in condensed matter physics, and so we address this problem.

On the basis of the Schauder fixed-point theorem, we first give another proof of the existence and uniqueness of the solution so as to show how the solution varies with the temperature. More precisely, we show that the solution belongs to a certain set, from which we see how the solution varies with the temperature.

On the basis of the Banach fixed-point theorem, we then show that the solution is indeed continuous with respect to both the temperature and the wavevector of an electron when the temperature satisfies a certain condition.

[1] Shuji Watanabe, *The solution to the BCS gap equation and the second-order phase transition in superconductivity*, J. Math. Anal. Appl. 383 (2011), 353-364.

[2] Shuji Watanabe, *Is the solution to the BCS gap equation continuous in the temperature ?*, arXiv: 1008.4436.

Sep–14 1630–1650

Entanglement of marginal tracial states

Hiromichi Ohno

Faculty of engineering, Shinshu University,
4-17-1 Wakasato, Nagano 380-8553, Japan
Email **h_ohno @shinshu-u.ac.jp**

States on coupled quantum systems have recently studied from many points of view. In this talk, we focus on states on the coupled quantum system $M_n(\mathbb{C}) \otimes M_n(\mathbb{C})$ whose restrictions to each subsystem are the normalized traces. Such states are called marginal tracial states.

Entanglement of an marginal tracial state is connected with the rank of the state. For example, if a marginal tracial state is pure, then the state is a maximally entangled state. And if the rank of an marginal tracial state is less than or equal to n, then the state is entangled.

So, we consider the maximal rank of extremal marginal tracial states. We will see that the maximal rank is bigger than n if $n \geq 3$, but an extremal marginal tracial state is entangled when the rank is $n + 1$.

[1] U. Haargerup and M. Musat, Factorization and dilation problems for completely positive maps on von Neumann algebras, Commun. Math. Phys. **303**, 555–594 (2011).

[2] M. Horodecki, P. W. Shor and M. B. Ruskai, Entanglement breaking channels, Rev. Math. Phys. **15**, 629–641 (2003).

[3] L. J. Landau and R. F. Streater, On Birkhoff's theorem for doubly stochastic completely positive maps on matrix algebras, Linear algebra appl. **193**, 107–127 (1993).

[4] H. Ohno, Maximal rank of extremal marginal tracial states, J. Math. Phys. **51**, 092101 (2010).

Existence of fixed points of firmly nonexpansive-like mappings in Banach spaces

Koji Aoyama

Department of Economics, Chiba University Yayoi-cho, Inage-ku, Chiba-shi, Chiba 263-8522, Japan
Email **aoyama @le.chiba-u.ac.jp**

The aim of this talk is to investigate a hybrid projection method and a hybrid shrinking projection method introduced in [6] for a single mapping of type (P) in a Banach space. Using the techniques in [2, 4, 5], we show that the sequences generated by these methods are well-defined without assuming the existence of fixed points. We also show that the boundedness of the generated sequences is equivalent to the existence of fixed points of mappings of type (P).

[1] K. Aoyama, F. Kohsaka, and W. Takahashi, *Three generalizations of firmly nonexpansive mappings: their relations and continuity properties*, J. Nonlinear Convex Anal. 10 (2009), 131–147.

[2] K. Aoyama, F. Kohsaka, and W. Takahashi, *Shrinking projection methods for firmly nonexpansive mappings*, Nonlinear Anal. 71 (2009), e1626–e1632.

[3] K. Aoyama and F. Kohsaka, *Existence of fixed points of firmly nonexpansive-like mappings in Banach spaces*, Fixed Point Theory Appl. 2010, Art. ID 512751, 15 pp.

[4] S. Matsushita and W. Takahashi, *A proximal-type algorithm by the hybrid method for maximal monotone operators in a Banach space*, in Nonlinear analysis and convex analysis, pp. 355–365, Yokohama Publ., Yokohama, 2007.

[5] S. Matsushita and W. Takahashi, *The sequences by the hybrid method and existence of fixed points of nonexpansive mappings in a Hilbert space*, in Fixed point theory and its applications, pp. 109–113, Yokohama Publ., Yokohama, 2008.

[6] W. Takahashi, Y. Takeuchi, and R. Kubota, *Strong convergence theorems by hybrid methods for families of nonexpansive mappings in Hilbert spaces*, J. Math. Anal. Appl. 341 (2008), 276–286.

Sep–14 1720–1740

Shrinking projection methods for common fixed point problems in a Banach space

Takanori Ibaraki

Department of General Science, Tsuruoka National College of Technology,
Sawada 104, Inooka, Tsuruoka, Yamagata 997-8511 Japan
Email **ibaraki @tsuruoka-nct.ac.jp**

In this talk, motivated by the result of Kimura and Takahashi [2] and that of Plubtieng and Ungchittrakool [3], we prove a strong convergence theorem for finding a common fixed point of generalized nonexpansive mappings in Banach spaces by using the shrinking projection method.

[1] T. Ibaraki and Y. Kimura, Convergence of nonlinear projections and shrinking projection methods for common fixed point problems, *Journal of Nonlinear Analysis and Optimization: Theory & Applications*, **2** (2011), 209-222.

[2] Y. Kimura and W. Takahashi, On a hybrid method for a family of relatively nonexpansive mappings in a Banach space, *J. Math. Anal. Appl.* **357** (2009), 356–363.

[3] S. Plubtieng and K. Ungchittrakool, Hybrid iterative methods for convex feasibility problems and fixed point problems of relatively nonexpansive mappings in Banach spaces, *Fixed Point Theory Appl.* **2008** (2008), Art. ID 583082, 19pp.

Sep–14 1740–1800

On Ishikawa's strong convergence theorem

Rieko Kubota and Yukio Takeuchi*

Yokohama So-gakukan High School,
1-43-1 Mutsuura-Higashi, Kanazawa, Yokohama 236-0037, Japan
Email **lri2e3ko_9 @yahoo.co.jp**

Takahashi Institute for Nonlinear Analysis,
1-11-11 Nakazato, Minami, Yokohama 232-0063, Japan
Email **aho31415 @yahoo.co.jp**

In 1976, Ishikawa proved the following theorem.

Theorem 1 (Ishikawa, 1976, [1]) *Let C be a closed convex subset of a real Banach space E, T be a nonexpansive self mapping on C such that $T(C)$ is relatively compact. Let b be a real number belonging to $(0,1)$ and let $\{\alpha_n\}$ be a sequence in $[0,b]$ with $\sum_{n=1}^{\infty}\alpha_n = \infty$. Let $\{u_n\}$ be a sequence defined by:*

$$u_1 \in C, \qquad u_{n+1} = \alpha_n T u_n + (1-\alpha_n)u_n \quad \text{for} \quad n \in N.$$

Then, $\{u_n\}$ converges strongly to a fixed point of T.

Before Ishikawa, Edelstein's theorem was known. In the theorem, strictly convexity of E is assumed. By shauder's theorem, his proof is based on the premise that a fixed point exists. Ishikawa removed strictly convexity of E and the premise. Then, Ishikawa's theorem is not merely a convergence theorem, but a constructive existence theorem in a general Banach space. To prove the theorem, Ishikawa prepared a lemma. Ishikawa's lemma is the main part of his proof. On the other hand, Suzuki [2] proved a useful lemma in 2002. Suzuki's lemma resembles Ishikawa's lemma in a way of thinking.

In this talk, we prove a lemma describing the structure that is common to these lemmas. Using the lemma, we can have Suzuki's lemma and versions of Ishikawa's lemma. Considering the process of Ishikawa's proof, we introduce some new families of mappings on a convex subset of a real Banach space. Requirement of the mappings is weaker than nonexpansiveness. For the mappings, we prove an extension of Ishikawa's strong convergence theorem.

[1] S. Ishikawa, *Fixed points and iteration of a nonexpansive mapping in a Banach space*, Proc. Amer. Math. Soc., **59** (1976), 65–71.

[2] T. Suzuki, *Strong convergence theorem to common fixed points of two nonexpansive mappings in general Banach spaces*, J. Nonlinear Convex Anal., **3** (2002), 381–391.

Metrizability of the Lévy topology on nonadditive measures

Jun Kawabe

Department of Mathematical Sciences, Shinshu University,
Wakasato, Nagano 380-8553, Japan
Email **jkawabe @shinshu-u.ac.jp**

Weak convergence of measures on a topological space not only plays a very important role in probability theory and statistics, but is also interesting from a topological measure theoretic view, since it gives a convergence closely related to the topology of the space on which the measures are defined. Thus, it is possible to study weak convergence of measures on a topological space in association with some topological properties of the space, such as the metrizability, separability and compactness.

Nonadditive measures, which are set functions that are monotonic and vanish at the empty set, have been extensively studied. They are closely related to nonadditive probability theory and the theory of capacities and random capacities. Nonadditive measures have been used in expected utility theory, game theory, and some economic topics under Knightian uncertainty.

The notion of weak convergence of nonadditive measures was formulated by Girotto and Holzer in a fairly abstract setting [1]. Some of their fundamental results for weak convergence, such as the portmanteau theorem and the direct and converse Prokhorov theorems, have been extended to the nonadditive case. In particular, the portmanteau theorem allows us to show that the weak topology, which is the topology generated by weak convergence, coincides with the Lévy topology, which is the topology generated by convergence of measures on a special class of sets.

In this talk, we will present successful nonadditive analogs of the theory of weak convergence of measures with a particular focus on metrizability and we will also supply weak convergence methods to related fields; see the author's recent paper [2].

[1] B. Girotto, S. Holzer, Weak convergence of bounded, monotone set functions in an abstract setting, Real Analysis Exchange 26 (2000/2001) 157-176.

[2] J. Kawabe, Metrizability of the Lévy topology on the space of nonadditive measures on metric spaces, Fuzzy Sets and Systems 204 (2012) 93-105.

Sep–15 1010–1040

Discrete fixed point theorems

Hidefumi Kawasaki

Faculty of Mathematics, Kyushu University,
Motooka 744, Nishi-ku, Fukuoka 819-0395, Japan
Email **kawasaki @ math.kyushu-u.ac.jp**

There are three types of discrete fixed point theorems: (M) monotone mapping, (C) contraction mappingws, and (B) reduced to Brouwer's fixed point theorem. In this talk, we compare discrete fixed point theorems by applying them to bimatrix games. Further, we show that simplical decompositions of the convex hull of the domain of the mapping are important for analyzing type (B).

We consider a bimatrix game with $m \times n$ payoff matrices $A = (a_{ij})$ and $B = (b_{ij})$. Players 1 and 2 maximize $x^T Ay$ and $x^T By$, respectively, where $x \in P_m$ and $y \in P_n$ are probability vectors. A pair of probability vectors $(\bar{x}, \bar{y})$ is called a *Nash equilibrium* if

$$x^T A\bar{y} \leq \bar{x}^T A\bar{y},\ \bar{x}^T By \leq \bar{x}^T B\bar{y} \quad \forall x \in P_m,\, y \in P_n.$$

In particular, when $\bar{x}$ and $\bar{y}$ are unit vectors e_i and e_j, respectively, $(\bar{x}, \bar{y})$ is called a *pure-strategy Nash equilibrium.* The set of best responses of each player is defined as follows: $F_1(j) = \{i \in \{1, \ldots, m\} \mid a_{ij} \geq a_{i'j}\ \forall i'\}$ and $F_2(i) = \{j \in \{1, \ldots, n\} \mid b_{ij} \geq b_{ij'}\ \forall j'\}$. Then a pure-strategy Nash equilibrium (e_i, e_j) is characterized by $(i, j) \in F(i, j) := F_1(j) \times F_2(i)$.

Let co$X$ be the convex hull of a finite set $X \subset Z^n$, and Σ a simplical decomposition of co$X$. We say two points $x, x' \in X$ to be *cell-connected* if they belong to a same simplex in Σ, and denote the binary relation $x \sim x'$. A mapping $f = (f_1, \ldots, f_n) : X \to X$ is said to be *direction preserving* if

$$x \sim x' \ \Rightarrow\ (f_i(x) - x_i)(f_i(x') - x'_i) \geq 0 \quad (i = 1, \ldots, n).$$

Iimura [1] proved that any direction preserving mapping has a fixed point. Key words of this talk are "simlical decomposition" and "direction preserving".

[1] T. Iimura, A discrete fixed point theorem and its applications, *J. Math. Econom.*, **39** (2003), 725–742.

Sep–15 1040–1110

Nonautonomous differential equations in Banach spaces with an application

Yoshikazu Kobayashi*, Naoki Tanaka and Yukino Tomizawa

Faculty of Science and Engineering, Chuo University
Email **kobayashi @ math.chuo-u.ac.jp**

Faculty of Science, Shizuoka University
Email **sntanak @ ipc.shizuoka.ac.jp**

Graduate School of Science and Engineering
Email **tomizawa @ gug.math.chuo-u.ac.jp**

Let X be a real Banach spces with norm $\|\cdot\|$. Let Ω be a subset of $[a, b) \times X (a < b \le \infty)$ sucn that $\Omega(t) = \{x\,;\,(t, x) \in \Omega\} \neq \emptyset$ and A a function defined on Ω into X. We consider the following initial-value problem for nonautonomous differential equation in X:

$$\begin{cases} u'(t) = A(t, u(t)) & \tau \le t < b, \\ u(\tau) = z, \end{cases} \qquad (\text{IVP};\tau, z)$$

where $(\tau, z) \in \Omega$. Our purpose is to establish an existence and uinqueness theorem for solutions of (IVP;τ, z) under the following general conditions (Ω2), (Ω3) and (Ω4) in addition to (Ω1) that A is continuous. Condition (Ω3) is called the subtangential condition and (Ω4) the dissipativity condition. Our dissipativity condition (Ω4) is so general that the usual norm-meric $\|x-y\|$ is replaced by the metric-like functional $V(t, x, y)$ which is defined for $(t, x, y) \in [a, b) \times X \times X$ and satisfies a several suitable conditions.

(Ω1) A is continuous from Ω into X.
(Ω2) If $(t_n, x_n) \in \Omega$, $t_n \uparrow t \in [a, b)$ and $x_n \to x$ in X as $n \to \infty$, then $(t, x) \in \Omega$.
(Ω3) $\liminf_{h \downarrow 0} \frac{1}{h} d(x + hA(t, x), \Omega(t + h)) = 0$ for all $(t, x) \in \Omega$.
(Ω4) $\liminf_{h \downarrow 0} \frac{1}{h} \big(V(t + h, x + hA(t, x), y + hA(t, y)) - V(t, x, y)\big) \le \omega(t)V(t, x, y)$ for all (t, x), $(t, y) \in \Omega$.

Here, ω be a real-valued continuous function on $[a, b)$ and $d(x, D)$ denote the distance from $x \in X$ to $D \subset X$. (Cf. Kenmochi-Takahashi(1980), Iwamiya(1982) and Kobayashi-Tanaka(2001).)

The results are applied to the initial value problem for a wave equation with damping term.

Some examples on p-uniform convexity and q-uniform smoothness

Tomonari Suzuki

Department of Mathematics, Kyushu Institute of Technology,
Tobata, Kitakyushu 804-8550, Japan
Email **suzuki-t @ mns.kyutech.ac.jp**

Let X be a *nontrivial* Banach space, by which mean a real Banach space with $\dim X \geq 2$, or a complex Banach space with $\dim X \geq 1$. The *modulus of convexity* of X is defined as

$$\delta(\varepsilon) = \inf \left(1 - \frac{\|x+y\|}{2} \right)$$

for $\varepsilon \in [0,2]$, where the infimum can be taken over all $x, y \in X$ with $\|x\| \leq 1$, $\|y\| \leq 1$ and $\|x-y\| \geq \varepsilon$. The *modulus of smoothness* of X is defined as

$$\rho(\tau) = \sup \left(\frac{\|x + \tau\, y\| + \|x - \tau\, y\|}{2} - 1 \right)$$

for $\tau \in (0,\infty)$, where the supremum can be taken over all $x, y \in X$ with $\|x\| \leq 1$ and $\|y\| \leq 1$. We know that if X is a Hilbert space, then $\delta(\varepsilon) = 1 - \sqrt{1 - \varepsilon^2/4}$ and $\rho(\tau) = \sqrt{1+\tau^2} - 1$.

We recall that X is said to be *uniformly convex* if $\delta(\varepsilon) > 0$ for all $\varepsilon > 0$. Also, X is said to be *uniformly smooth* if $\lim_{\tau \to +0} \rho(\tau)/\tau = 0$.

For $p \in [2,\infty)$, X is called p-uniformly convex if there exists $C > 0$ satisfying

$$\delta(\varepsilon) \geq C\, \varepsilon^p$$

for all $\varepsilon \in [0,2]$. On the other hand, for $q \in (1,2]$, X is called q-uniformly smooth if there exists $K > 0$ satisfying

$$\rho(\tau) \leq K\, \tau^q$$

for all $\tau \in (0,\infty)$. It is obvious that p-uniformly convex Banach spaces are uniformly convex, and q-uniformly smooth Banach spaces are uniformly smooth. We also know that for $p \in (1,\infty)$, L^p spaces are $\max\{2,p\}$-uniformly convex and $\min\{2,p\}$-uniformly smooth.

In our talk, we give some examples of 2-dimensional real Banach spaces. and discuss p-uniform convexity and q-uniform smoothness.

Convolution operators and div-curl lemma on weighted Hardy sapces with an application to Navier-Stokes equations

Yohei Tsutsui

Department of Mathematics, School of Science and Engineering,
Waseda University, Shinjuku, TOKYO 160, JAPAN
Email **y.tsutsui @kurenai.waseda.jp**

In this talk, we consider the mapping properties of convolution operators with smooth functions and esbalish so-called div-curl lemma on weighted Hardy spaces $H^p(w)$ with w belonging to Muckenhoupt weight class A_∞. The proof of the boundedness of convolution operators uses atomic decompositions by García-Cuerva [3] and Strömberg-Torchinsky [7], and molecular characterization by Taibleson-Weiss [8] and Lee-Lin [5]. From pointwise estimates for the garand maximal functions due to Miyachi [6] in the non-endpoint case and the approach due to Auscher-Russ-Tchamitchian [1] in the endpoint case, our div-curl lemma is proved. Applying these estimates, we investigate the time decay property of solutions to incompressible Navier-Stokes equations in whole space $\mathbb{R}^n$ with $n \geq 2$

$$\text{(N-S)} \quad \begin{cases} \partial_t u - \Delta u + (u \cdot \nabla) u + \nabla p = 0, \\ \text{div } u = 0, \\ u(0) = a. \end{cases}$$

In particular, we are interested in the decay property of Kato's [4] global solutions when the initial data a belongs to weighted Hardy spaces. For example, our decay order of energy of the solutions $\|u(t)\|_{L^2}$ can be close to the critical one $(n+2)/4$ in the sense of Wiegner [9], as possible.

Our first result related to the mapping property of convolution operators reads as follows.

Theorem 1 *Let $0 < p \leq q < \infty$ and $w, \sigma \in A_\infty$. If there exists $K > 0$ such that*

$$[w, \sigma]_{X^K_{p,q}} = \sup_B \min\left(1, |B|^K\right) \frac{\sigma(B)^{1/q}}{w(B)^{1/p}} < \infty,$$

where the supremum is taken over all balls B, then for any $\varphi \in \mathcal{S}$ we have

$$\|f * \varphi\|_{H^q(\sigma)} \leq c[w, \sigma]_{X^K_{p,q}} \|f\|_{H^p(w)}$$

where the constant c depends on $p, q, n, \varphi, [w]_{A_\infty}$ and $[\sigma]_{A_\infty}$.

after div-curl lemma on weighted Hardy spaces is established, we research the time decay of solutions to the Navier-Stokes equation and obtain the following.

Theorem 2 *Let* $1 \le p < \infty$, $-n/p < \alpha < n(1-1/p)+1$ *and* $w(x) = |x|^{\alpha p} \in A_{p(1+1/n)}$. *Then, there exists* $\delta > 0$ *such that for any* $a \in L^n \cap H^p(w)$ *with* $\|a\|_{L^n} \le \delta$ *and div* $a = 0$, *we can construct a solution* $u \in L^\infty(0,\infty; L^n \cap H^p(w)) \cap C((0,\infty); L^n \cap H^p(w)) \cap C^\infty((0,\infty)\times\mathbb{R}^n)$ *of* (I.E.) *satisfying*

$$\begin{aligned} &\sup_{t>0} \|u(t)\|_{H^p(w)} \le c\|a\|_{H^p(w)} \\ &\lim_{t\searrow 0} \|u(t)-a\|_{L^n} = \lim_{t\searrow 0} \|u(t) - e^{t\Delta}a\|_{H^p(w)} = 0 \\ &\sup_{t>0} t^{-1/2}\|\nabla u(t)\|_{H^p(w)} < \infty. \end{aligned}$$

Moreover, for $q \in [p,\infty)$ *and* $\beta \in (-n/q, n(1-1/p)+1)$ *with* $\beta \le \alpha$, *the solution* u *satisfies the following decay property;*

$$\|u(t)\|_{H^q(\sigma)} \lesssim t^{-n(1/p-1/q)/2-(\alpha-\beta)/2}\|a\|_{H^p(w)}, \tag{1}$$

where $\sigma(x) = |x|^{\beta q} \in A_{q(1+1/n)}$. *In particular, in the case* $p < q$ *or* $\beta < \alpha$, *it holds that*

$$\|u(t)||_{H^q(\sigma)} = o(t^{-\gamma}) \quad \text{as } t \searrow 0, \tag{2}$$

where $\gamma = n(1/p-1/q)/2 + (\alpha-\beta)/2$.

[1] P. Auscher, E. Russ and P. Tchamitchian, *Hardy Sobolev spaces on strongly Lipschitz domains of* $\mathbb{R}^n$, J. Funct. Anal. **218** (2005), no. 1, 54-109.

[2] R. Coifman, P.L. Lions, Y. Meyer and S. Semmes, *Compensated compactness and Hardy spaces*, J. Math. Pures Appl. **72** (1993), 247-286.

[3] J. García-Cuerva, *Weighted* H^p*-spaces*, Disertationes Math. **162** (1979).

[4] T. Kato, *Strong* L^p*-solutions of the Navier-Stokes equation in* $\mathbb{R}^n$*, with applications to weak solutions*, Math. Z. **187** (1984), 471-480.

[5] M.-Y. Lee and C.-C. Lin, *The molecule characterization of weighted Hardy spaces*, J. Funct. Anal. **188** (2002), 442-460.

[6] A. Miyachi, *Hardy space estimate for the product of singular integrals*, Canad. J. Math. **52** (2000), no.2, 381-411.

[7] J.-O. Strömberg and A. Torchinsky, *Weighted Hardy spaces*, Lecture Notes in Mathematics, **1381**. Springer-Verlag, Berlin, (1989).

[8] M.H. Taibleson and G. Weiss, *The molecular characterization of certain Hardy spaces. Representation theorems for Hardy spaces*, Astérisque, **77**, Soc. Math. France, Paris, (1980) 67-149.

[9] M. Wiegner, *Decay results for weak solutions of the Navier-Stokes equations on* $\mathbb{R}^n$, J. London Math. Soc. (2) **35** (1987), no. 2, 303-313.

Sep–15 1210–1230

New Notions of Differential Calculus of L^p-functions and L^p_{loc}-functions

Yoshifumi Ito

209-15 Kamifukuman Hachiman-cho,
Tokushima 770-8073, Japan
Email **yoshifumi @ md.pikara.ne.jp**

In this paper, we study the new notions and the fundamental properties of derivatives and partial derivatives of L^p-functions and L^p_{loc}-functions in the sense of L^p-convergence and L^p_{loc}-convergence respectively.

Here we assume that p is a real number such that $1 \leq p < \infty$ holds.

In the calculation of such derivatives and partial derivatives, we do not need the theory of distributions except the case $p = 1$.

Thereby, I give the new characterization of Sobolev spaces and give the new meaning of Stone's Theorem.

Especially, in the cases of L^2-functions and L^2_{loc}-functions, these results have the essential role in the study of Schrödinger equations.

Every solution of Schrödinger equation which is the basic equation of natural statistical physics should be an L^2-density. Further, the solutions of an eigenvalue problem of a Schrödinger operator can be obtained as L^2-densities or L^2_{loc}-densities.

For these functions, these derivatives in the sense of distributions are the most general one at present.

But, when we consider the derivatives of L^2-functions and L^2_{loc}-functions, it is extreme to consider the derivatives in the sense of distributions.

It is not easy to judge that the derivatives or the partial derivatives of L^2-functions in the sense of distributions are L^2-functions.

Therefore, it is hard to see the concrete meanings of the concept of derivatives in the sense of distributions.

Against the above, if we calculate the derivatives or the partial derivatives of L^2-functions in the sense of L^2-convergence, the results are L^2-functions at once.

Similarly, if we calculate the derivatives or the partial derivatives of L^2_{loc}-functions in the sense of L^2_{loc}-convergence, the results are L^2_{loc}-functions at once. Therefore, the differential calculus in the sense of L^2-convergence or L^2_{loc}-convergence is considered to be very concrete and useful.

Therefore, for the differential equations such as Schrödinger equations, the concepts of differential calculus considered here are very useful.

INDEX