The Fifth International Symposium on

Banach and Function Spaces 2015

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September 2 (Wednesday)

0900–0950	Registration
0950–1000	Opening
1000–1040	Gilles Godefroy Universally measurable functions and set theory
1050–1130	Wataru Takahashi Weak and strong convergence theorems for split common null point problems in Banach spaces and applications
1140–1210	Yasunori Kimura Iterative schemes with nonvanishing errors for common fixed point problems
1210–1240	Toshitaka Nagai On the Cauchy problem of a parabolic-elliptic Keller-Segel equation in $\mathbb{R}^2$ with L^1 initial data
1420–1440	Paweł Kolwicz Pointwise multipliers and products of Banach function spaces
1440–1500	Karol Leśnik Interpolation of Cesàro and Tandori spaces
1500–1520	Radosław Kaczmarek Ellipsoidal geometry of Banach spaces and applications
1540–1600	Jarno Talponen Varying exponent L^p norms defined by weak solutions to ODEs
1600–1620	Shuichi Sato Square functions related to Marcinkiewicz integral and Sobolev spaces
1620–1640	Wei-Shih Du On new generalizations of Smarzewski's fixed point theorem
1700–1720	Fumiaki Kohsaka Asymptotic behavior of averaged sequences for nonspreading mappings in Banach spaces
1720–1740	Sachiko Atsushiba Attractive points and fixed point properties for nonlinear mappings
1900–2100	Welcome Party (Kokura Recent Hotel)

September 3 (Thursday)

0930–1010	Michael Cwikel Some new partial answers to a 52 year old interpolation question
1020–1100	David Yost Linear extensions and their applications
1120–1150	Ryszard Płuciennik Local Kadec-Klee properties in symmetric function spaces
1150–1220	Kichi-Suke Saito*, Naoto Komuro and Ryotaro Tanaka Which Banach space has James constant $\sqrt{2}$?
1400–1420	Alexei Yu. Karlovich The Stechkin inequality for Fourier multipliers on Nakano spaces
1420–1440	Takayuki Tamura*, Sompong Dhompongsa and Mikio Kato On A-direct sums of Banach spaces
1440–1500	Paweł Foralewski M-constants in some Banach lattices
1520–1540	Ken-Ichi Mitani*, Yasuji Takahashi and Kichi-Suke Saito On the von Neumann-Jordan constant for Banaś-Frączek space
1540–1600	Ryotaro Tanaka* and Kichi-Suke Saito The duality equality for James constant of Banach spaces
1600–1620	Mayumi Hojo* and Wataru Takahashi Fixed point and weak convergence theorems for nonlinear hybrid mappings in Banach spaces
1640–1700	Tomonari Suzuki Topology on ν-generalized metric spaces
1700–1720	Yoshifumi Ito Fourier transformation of L^2_{loc}-functions
1720–1740	Waichiro Matsumoto Separativity of ultradifferentiable classes

September 4 (Friday)

0930–1010	Kazimierz Goebel Trend constants for Lipschitz mappings
1020–1100	Lixin Cheng A universal theorem for stability of almost isometries
1120–1150	Yoshihiro Sawano* and Denny Ivanal Hakim Complex interpolation of Morrey spaces–second method
1150–1220	Noriaki Suzuki Carleson inequalities for $L^{(\alpha)}$-harmonic functions
1220–1230	Photo Shoot
1400–1420	Takuya Sobukawa B^u_w-function spaces and their interpolation
1420–1440	Hang-Chin Lai On $A^p(G)$-algebras, $1 \leq p < \infty$ and the multipliers of $A^p(G)$ for $1 \leq p \leq 2$
1440–1500	Shuechin Huang* and Yasunori Kimura A projection method for approximating fixed points of quasinonexpansive mappings in complete metric spaces
1520–1540	Ing-Jer Lin* and Yan-Wei Wu Some new convergence theorems for new nonlinear cyclic mappings on quasiordered metric spaces
1540–1600	Hideyuki Wada* and Yasunori Kimura An iterative scheme extending Halpern type in a Hadamard space
1600–1620	Pongsakorn Yotkaew*, Yasunori Kimura and Satit Saejung The Mann algorithm in a complete geodesic space with curvature bounded above
1620–1640	Toshiharu Kawasaki Fixed point theorems for widely more generalized hybrid mappings in a Banach space
1640–1700	Yukio Takeuchi On Ishikawa's convergence theorem for pseudo-contractions
1800–2030	Memorial Concert at Kitakyushu Geijutsu-Gekijo (Violin Recital)

September 5 (Saturday)

0930–1010	Gord Sinnamon* and Wayne Grey Mixed-norm Lebesgue spaces: Inequalities and inclusions
1020–1100	Thomas Kühn Approximation of multivariate periodic Sobolev functions
1120–1150	Suthep Suantai On Browder's convergence theorem and Halpern iteration process for G-nonexpansive mappings in Hilbert spaces endowed with graphs
1150–1220	Kenjiro Yanagi Generalized quasi-metric adjusted skew information and trace inequality
1400–1420	Jun Kawabe Bounded convergence theorems for nonlinear integrals
1420–1440	Ryskul Oinarov* and Aigerim Kalybay Spaces with multiweighted derivatives
1440–1500	Osamu Hatori Examples of generalized gyrovector spaces
1520–1540	Narawadee Na Nan Coupled fixed point theorems for $\alpha - \psi -$Geraghty's contraction maps using monotone property
1540–1600	Boris S. Mordukhovich and Nobusumi Sagara* Subdifferentials of nonconvex integral functionals in Banach spaces: A Gelfand integral representation
1600–1620	Shogo Kobayashi, Yutaka Saito, Tamaki Tanaka* and Syuuji Yamada Study on set-valued inequality based on set-valued analysis and covenx analysis
1640–1700	Yukino Tomizawa Asymptotically quasi-nonexpansive mappings with respect to the Bregman distance
1700–1720	Takanori Ibaraki Shrinking projection methods with error for zero point problems in a Hilbet space
1720–1740	Yi-Chou Chen Upper bound of the number of all solutions for integer polynomial equations (modulo p^m)
1900–2100	Banquet at Hotel New Tagawa

September 6 (Sunday)

0930–1010	Lech Maligranda Structure of Cesàro function spaces and interpolation
1020–1100	Eiichi Nakai Pointwise multipliers on several function spaces
1120–1150	Yoshihiro Mizuta Sobolev's inequality for Riesz potentials of functions in central Herz-Morrey-Orlicz spaces on the unit ball
1150–1220	Marek Wisła Monotonicity properties of Orlicz spaces equipped with the p-Amemiya norm
1230–1740	Short Excursion (Kamon Wharf, Shimonoseki Sake Maker, Akama Shrine, Kokura Castle)

Sep–2 1000–1040

Universally measurable functions and set theory

Gilles Godefroy

Centre National de la Recherche Scientifique - University Paris 6
Email **gilles.godefroy @ imj-prg.fr**

Let (f_n) be a uniformly bounded sequence of continuous functions on a compact metric space. We denote by C_k the pointwise closure of the convex hull of the set $(f_n)_{n>k}$. Does the intersection of the sequence (C_k) contain a universally measurable function? It turns out that the answer to this question depends upon the axioms of set theory. We will display the (recent) work of P. Larson and the (not so recent) works of J.P.R. Christensen and G. Mokobodzki on these matters, and present some simplified proofs and related results.

Weak and strong convergence theorems for split common null point problems in Banach spaces and applications

Wataru Takahashi

Keio Research and Education Center for Natural Sciences, Keio University, Kouhoku-ku, Yokohama 223-8521, Japan
Email **wataru @is.titech.ac.jp; wataru@a00.itscom.net**

Let H_1 and H_2 be two real Hilbert spaces. Let D and Q be nonempty, closed and convex subsets of H_1 and H_2, respectively. Let $A : H_1 \to H_2$ be a bounded linear operator. Then the *split feasibility problem* is to find $z \in H_1$ such that $z \in D \cap A^{-1}Q$. Recently, Byrne, Censor, Gibali and Reich also considered the following problem: Given set-valued mappings $A_i : H_1 \to 2^{H_1},\ 1 \le i \le m$, and $B_j : H_2 \to 2^{H_2},\ 1 \le j \le n$, respectively, and bounded linear operators $T_j : H_1 \to H_2,\ 1 \le j \le n$, the *split common null point problem* is to find a point $z \in H_1$ such that

$$z \in \left(\bigcap_{i=1}^{m} A_i^{-1}0 \right) \cap \left(\bigcap_{j=1}^{n} T_j^{-1}(B_j^{-1}0) \right),$$

where $A_i^{-1}0$ and $B_j^{-1}0$ are null point sets of A_i and B_j, respectively. Defining $U = A^*(I - P_Q)A$ in the split feasibility problem, we have that $U : H_1 \to H_1$ is an inverse strongly monotone operator, where A^* is the adjoint operator of A and P_Q is the metric projection of H_2 onto Q. Furthermore, if $D \cap A^{-1}Q$ is nonempty, then $z \in D \cap A^{-1}Q$ is equivalent to

$$z = P_D(I - \lambda A^*(I - P_Q)A)z, \tag{1}$$

where $\lambda > 0$ and P_D is the metric projection of H_1 onto D. Using such results regarding nonlinear operators and fixed points, many authors have studied the split feasibility problem and the split common null point problem in Hilbert spaces.

In this talk, motivated by iterative methods for split feasibility problems and split common null point problems in Hilbert spaces, we consider such problems in Banach spaces. Then, using geometry of Banach spaces, we establish weak and strong convergence theorems for split feasibility problems and split common null point problems in Banach spaces. It seems that such theorems are first in Banach spaces. Furthermore, using these results, we get well-known and new results which are connected with split feasibility problems and split common null point problems.

Iterative schemes with nonvanishing errors for common fixed point problems

Yasunori Kimura

Department of Information Science, Toho University,
2-2-1, Miyama, Funabashi, Chiba 274-8510, Japan
Email **yasunori @ is.sci.toho-u.ac.jp**

Let C be a nonempty closed convex subset of a real Banach space. We consider the common fixed point problem for nonexpansive mappings $T_i : C \to C$, which is defined as to find a point z point $z \in C$ satisfying that $z = T_i z$ for every $i \in I$. This simple problem has been applied to various types of nonlinear problems such as convex minimization problems, equilibrium problems, variational inequality problems, saddle point problems, and others.

In 2008, Takahashi, Takeuchi, and Kubota [3] established a strong convergence theorem by a new type of projection method, which is also called the shrinking projection method.

In this talk, we study an iterative scheme generated by the shrinking projection method with calculation errors. In practical calculation, it is difficult to find the exact values of metric projections which appear in iterative sequence by this method. To overcome this difficulty, we consider an error for obtaining the value of metric projections and show that the sequence still has a tolerable property for approximating a common fixed point of the family of mappings. Notice that we do not assume that the error sequence vanishes. To prove the main result, we employ the techniques developed in [1, 2].

[1] Y. Kimura, *Approximation of a fixed point of nonlinear mappings with nonsummable errors in a Banach space*, Proceedings of the Fourth International Symposium on Banach and Function Spaces, 2012, pp. 303–311.

[2] Y. Kimura, *Approximation of a common fixed point of a finite family of nonexpansive mappings with nonsummable errors in a Hilbert space*, J. Nonlinear Convex Anal. **15** (2014), 429–436.

[3] W. Takahashi, Y. Takeuchi, and R. Kubota, *Strong convergence theorems by hybrid methods for families of nonexpansive mappings in Hilbert spaces*, J. Math. Anal. Appl. **341** (2008), 276–286.

On the Cauchy problem of a parabolic-elliptic Keller-Segel equation in $\mathbb{R}^2$ with L^1 initial data

Toshitaka Nagai

Hiroshima University,
Higashi-Hiroshima 739-8526, Japan
Email **nagai @ math.sci.hiroshima-u.ac.jp**

In this talk, we consider the Cauchy problem of the following parabolic-elliptic system in $\mathbb{R}^2$:

$$\begin{cases} \partial_t u = \Delta u - \nabla \cdot (u \nabla \psi), & t > 0,\ x \in \mathbb{R}^2, \\ -\Delta \psi = u, & t > 0,\ x \in \mathbb{R}^2, \\ u|_{t=0} = u_0, & x \in \mathbb{R}^2, \end{cases}$$

where ψ is specified by

$$\psi(t,x) = (N * u)(t,x) := \int_{\mathbb{R}^2} N(x-y) u(t,y)\, dy,$$
$$N(x) = -\frac{1}{2\pi} \log |x| \quad \text{(the logarithmic potential)},$$

and the initial data u_0 is assumed to be in $L^1(\mathbb{R}^2)$ and nonnegative on $\mathbb{R}^2$. The system is a simplified version derived from the original parabolic Keller-Segel equation of chemotaxis (the movement of biological cells or organisms in response to chemical gradients), and also a mathematical model describing the movement of the gravitational particles in astronomy.

The total mass of the nonnegative solutions to the Cauchy problem is conserved, namely, $\int_{\mathbb{R}^2} u(t,x)\, dx = \int_{\mathbb{R}^2} u_0\, dx$, and the global existence and large-time behavior of the solutions heavily depend on the total mass of the initial data $M := \int_{\mathbb{R}^2} u_0\, dx$. It is known that a nonnegative solution may blow up in finite time if $M > 8\pi$(supercritical case), on the other hand, every nonnegative solution exists globally in time and decays to zero as time goes to infinity if $M < 8\pi$(subcritical case).

The local existence of solutions in time is obtained by applying the fixed point theorem for a contraction mapping in a function space. In the critical case $M = 8\pi$, we mention the global existence of nonnegative solutions in time under mild conditions on the initial data.

Sep–2 1420–1440

Pointwise multipliers and products of Banach function spaces

Paweł Kolwicz

Institute of Mathematics, Poznań University of Technology,
Poznań, Poland
Email : **pawel.kolwicz @ put.poznan.pl**

The well-known factorization theorem of Lozanovskiĭ may be written in the form $L^1 \equiv E \odot E'$, where $\odot$ means the pointwise product of Banach ideal spaces and E' is a Köthe dual of E. A natural generalization of this problem would be the question when one can factorize F through E, i.e., when

$$F \equiv E \odot M(E, F)\,, \tag{1}$$

where $M(E, F)$ is the space of pointwise multipliers from E to F. Given two Banach ideal spaces E and F on a measure space (Ω, Σ, μ) define the pointwise product space $E \odot F$ as

$$E \odot F = \{x \cdot y : x \in E \text{ and } y \in F\}\,.$$

with a functional $\|\cdot\|_{E \odot F}$ defined by the formula

$$\|z\|_{E \odot F} = \inf\left\{\|x\|_E\, \|y\|_F : z = xy, x \in E, y \in F\right\}. \tag{2}$$

Moreover, the space of multipliers $M(E, F)$ is defined as

$$M(E, F) = \left\{x \in L^0 : xy \in F \quad \text{for} \quad \text{each} \quad y \in E\right\}$$

with the operator (semi) norm

$$\|x\|_{M(E,F)} = \sup_{\|y\|_E = 1} \|xy\|_F\,.$$

We will discuss some basic properties of the constructions $M(E, F)$ and $E \odot F$. Next we will consider the equality (1) just with the equivalence of norms, that is, $F = E \odot M(E, F)$, which also seems to be useful. We present results concerning such factorization in selected classes of Banach function spaces. This can be done by finding $M(E, F)$ and $E \odot F$ separately. Thus the form of spaces $M(E, F)$ and $E \odot F$ for concrete class of Banach spaces may be applied at this point.

The talk is based on the papers [1, 2].

[1] P. Kolwicz, K. Leśnik and L. Maligranda, Pointwise multipliers of Calderón-Lozanovskiĭ spaces, Math. Nachr. 286 (8-9) (2013), 876–907.

[2] P. Kolwicz, K. Leśnik and L. Maligranda, Pointwise products of some Banach function spaces and factorization, J. Funct. Anal. 266 (2) (2014), 616-659.

Interpolation of Cesàro and Tandori spaces

Karol Leśnik

Institute of Mathematics, Poznań University of Technology,
Poznań, Poland
Email **klesnik @ vp.pl**

We will discuss some recent developments on interpolation of abstract Cesàro spaces CX and Tandori spaces $\widetilde{X}$, where for a given Banach function space X on $I = (0, 1)$ or $I = (0, \infty)$

$$CX = \{f \in L^0 : (\frac{1}{x}\int_0^x |f|)_{x \in I} \in X\},$$

$$\widetilde{X} = \{f \in L^0 : (\operatorname*{ess\,sup}_{t \geq x} |f(t)|)_{x \in I} \in X\}.$$

Firstly we will explain how the monotone version of Hardy-Littlewood-Pólya submajorization theorem together with monotone substochastic operators leads to conclusion that $(\widetilde{L^1}, L^\infty)$ is a Calderón couple, which answers in positive the question of Sinnamon. In the second part we shall present results on the real and complex interpolation of these kind of spaces.

[1] K. Leśnik and L. Maligranda, *On abstract Cesàro spaces. Duality*, J. Math. Anal. Appl. 424 (2015), 932–951.

[2] K. Leśnik, *Monotone substochastic operators and a new Calderón couple*, Studia Math. 227 (2015), 21–39.

[3] K. Leśnik and L. Maligranda, *Interpolation of abstract Cesàro, Copson and Tandori spaces*, preprint available at: http://arxiv.org/pdf/1502.05732.pdf.

Sep–2 1500–1520

Ellipsoidal geometry of Banach spaces and applications

Radosław Kaczmarek

Adam Mickiewicz University in Poznań
Faculty of Mathematics and Computer Science,
Poznań, Poland
Email **radekk @amu.edu.pl**

The notions of strict convexity, uniform convexity, non-squareness and uniform non-squareness of Banach spaces in the ellipsoidal sense (called shortly ellipsoidal properties) will be presented and considered. One will show that any of these properties means its classical counterpart for a family of equivalent norms being Minkowski functional of ellipsoids $\{x \in X : \|x+y_0\|+\|x-y_0\| \leq c\}$ with a fixed norm $0 < \|y_0\| < \frac{c}{2}$ and all $c > 0$ or with a fixed $c > 0$ and all y_0 with $0 < \|y_0\| < \frac{c}{2}$. Finally, it will be shown that ellipsoidally uniformly convex Banach spaces enjoy the Banach-Saks property as well as that in any ellipsoidally uniformly convex Banach space X the ellipsoids $E_c(y_0)$ with $\|y_0\| \in \left(0, \frac{c}{2}\right)$ are approximatively compact Chebyshev sets.

Sep–2 1540–1600

Varying exponent L^p norms defined by weak solutions to ODEs

Jarno Talponen

University of Eastern Finland,
Yliopistokatu 7, Metria M230, Joensuu, Finland
Email **talponen @ iki.fi**

We will discuss a new way of defining a varying exponent $L^{p(\cdot)}$-norm, thus completely different from the function spaces carrying the names of Luxemburg, Musielak and Orlicz. This function space norm is defined by using Caratheodory's weak solutions to a suitable ordinary differential equation, which encode the values of the absolute value of the function and the exponent. The resulting class contains as special cases the classical L^p spaces.

Sep–2 1600–1620

Square functions related to Marcinkiewicz integral and Sobolev spaces

Shuichi Sato

Department of Mathematics, Faculty of Education,
Kanazawa University, Kanazawa 920-1192, Japan
Email **shuichi @kenroku.kanazawa-u.ac.jp**

Let ψ be a function in $L^1(\mathbb{R}^n)$ satisfying

$$\int_{\mathbb{R}^n} \psi(x)\, dx = 0.$$

We consider the Littlewood-Paley function on $\mathbb{R}^n$ defined by

$$g_\psi(f)(x) = \left(\int_0^\infty |f * \psi_t(x)|^2 \frac{dt}{t} \right)^{1/2},$$

where $\psi_t(x) = t^{-n}\psi(t^{-1}x)$, and a discrete parameter version of g_ψ:

$$\Delta_\psi(f)(x) = \left(\sum_{k=-\infty}^{\infty} |f * \psi_{2^k}(x)|^2 \right)^{1/2}.$$

Theorem 1. *Suppose that*

(1) $B_\epsilon(\psi) < \infty$ *for some* $\epsilon > 0$, *where* $B_\epsilon(\psi) = \int_{|x|>1} |\psi(x)|\, |x|^\epsilon\, dx$;

(2) $D_u(\psi) < \infty$ *for some* $u > 1$ *with* $D_u(\psi) = \left(\int_{|x|<1} |\psi(x)|^u\, dx \right)^{1/u}$;

(3) $H_\psi \in L^1(\mathbb{R}^n)$, *where* $H_\psi(x) = \sup_{|y| \ge |x|} |\psi(y)|$;

(4) $m(\xi) = \int_0^\infty |\hat{\psi}(t\xi)|^2\, dt/t \neq 0$ *for all* $\xi \neq 0$.

Then $\|f\|_{p,w} \simeq \|g_\psi(f)\|_{p,w}$, $f \in L^p_w$, *for all* $p \in (1,\infty)$ *and* $w \in A_p$ *(the Muckenhoupt class).*

Theorem 2. *We assume that*

(1) $B_\epsilon(\psi) < \infty$ *for some* $\epsilon > 0$;

(2) $|\hat{\psi}(\xi)| \le C|\xi|^{-\delta}$ *for all* $\xi \in \mathbb{R}^n \setminus \{0\}$ *with some* $\delta > 0$;

(3) $H_\psi \in L^1(\mathbb{R}^n)$;

(4) $m(\xi) = \sum_{k=-\infty}^{\infty} |\hat{\psi}(2^k\xi)|^2 \neq 0$ *for all* $\xi \neq 0$.

Then $\|f\|_{p,w} \simeq \|\Delta_\psi(f)\|_{p,w}$, $f \in L^p_w$, *for all* $p \in (1, \infty)$ *and* $w \in A_p$.

We apply Theorems 1, 2 to characterize weighted Sobolev spaces $W^{\alpha,p}_w(\mathbb{R}^n)$ by square functions related to the Marcinkiewicz integral including

$$\left(\int_0^\infty \left| f(x) - \fint_{B(x,t)} f(y)\, dy \right|^2 \frac{dt}{t^{1+2\alpha}}\right)^{1/2}, \quad \alpha > 0,$$

$$\left(\sum_{k=-\infty}^{\infty} \left| f(x) - \fint_{B(x,2^k)} f(y)\, dy \right|^2 2^{-2k\alpha}\right)^{1/2}, \quad \alpha > 0,$$

where $\fint_{B(x,t)} f(y)\, dy = |B(x,t)|^{-1} \int_{B(x,t)} f(y)\, dy$ and $|B(x,t)|$ is the Lebesgue measure of a ball $B(x,t)$ in $\mathbb{R}^n$ with center x and radius t.

We say $\Phi \in \mathcal{M}^\alpha$, $\alpha > 0$, if Φ is a compactly supported, bounded function on $\mathbb{R}^n$ satisfying $\int_{\mathbb{R}^n} \Phi(x)\, dx = 1$; if $\alpha \geq 1$, we further assume that

$$\int_{\mathbb{R}^n} \Phi(x) x^\gamma\, dx = 0, \quad x^\gamma = x_1^{\gamma_1} \dots x_n^{\gamma_n}, \quad \text{for all } \gamma \text{ with } 1 \leq |\gamma| \leq [\alpha].$$

Let

$$U_\alpha(f)(x) = \left(\int_0^\infty |f(x) - \Phi_t * f(x)|^2 \frac{dt}{t^{1+2\alpha}}\right)^{1/2}, \quad \alpha > 0,$$

$$E_\alpha(f)(x) = \left(\sum_{k=-\infty}^{\infty} |f(x) - \Phi_{2^k} * f(x)|^2\, 2^{-2k\alpha}\right)^{1/2}, \quad \alpha > 0,$$

with $\Phi \in \mathcal{M}^\alpha$.

Then we have the following results.

Theorem 3. *Let* $1 < p < \infty$, $w \in A_p$ *and* $0 < \alpha < n$. *Then* $f \in W^{\alpha,p}_w(\mathbb{R}^n)$ *if and only if* $f \in L^p_w$ *and* $U_\alpha(f) \in L^p_w$; *furthermore,*

$$\|f\|_{p,\alpha,w} \simeq \|f\|_{p,w} + \|U_\alpha(f)\|_{p,w},$$

where $\|f\|_{p,\alpha,w}$ *denotes the norm in* $W^{\alpha,p}_w(\mathbb{R}^n)$.

Theorem 4. *Suppose that* $1 < p < \infty$, $w \in A_p$ *and* $0 < \alpha < n$. *Then* $f \in W^{\alpha,p}_w(\mathbb{R}^n)$ *if and only if* $f \in L^p_w$ *and* $E_\alpha(f) \in L^p_w$; *also,*

$$\|f\|_{p,\alpha,w} \simeq \|f\|_{p,w} + \|E_\alpha(f)\|_{p,w}.$$

[1] R. Alabern, J. Mateu and J. Verdera, *A new characterization of Sobolev spaces on* $\mathbb{R}^n$, Math. Ann. **354** (2012), 589–626.

[2] P. Hajłasz, Z. Liu, *A Marcinkiewicz integral type characterization of the Sobolev space*, arXiv:1405.6127 [math.FA].

[3] S. Sato, *Littlewood-Paley equivalence and homogeneous Fourier multipliers*, preprint (2015).

Sep–2 1620–1640

On new generalizations of Smarzewski's fixed point theorem

Wei-Shih Du

Department of Mathematics, National Kaohsiung Normal University,
Kaohsiung 82444, Taiwan
Email **wsdu @ nknucc.nknu.edu.tw, wsdu@mail.nknu.edu.tw**

In this work, we prove some new generalizations of Smarzewski's fixed point theorem and some new fixed point theorems which are original and quite different from the well known results in the literature.

Sep–2 1700–1720

Asymptotic behavior of averaged sequences for nonspreading mappings in Banach spaces

Fumiaki Kohsaka

Department of Mathematical Sciences, Tokai University,
Kitakaname, Hiratsuka-shi, Kanagawa 259-1292, Japan
Email **f-kohsaka @ tsc.u-tokai.ac.jp**

We show that the norm of an averaged iterative sequence for a nonspreading mapping in a Banach space is divergent to infinity if and only if the mapping has no fixed point. Using this result, we study the existence of a zero point of a monotone operator in a Banach space.

Let C be a nonempty closed convex subset of a smooth Banach space X and J the normalized duality mapping of X into X^*. A mapping $T\colon C \to X$ is said to be nonspreading [2] if

$$\phi(Tx, Ty) + \phi(Ty, Tx) \leq \phi(Tx, y) + \phi(Ty, x)$$

for all $x, y \in C$, where ϕ is the function of $X \times X$ into $[0, \infty)$ defined by

$$\phi(u, v) = \|u\|^2 - 2\langle u, Jv\rangle + \|v\|^2$$

for all $u, v \in X$.

The main result obtained in this talk is stated below:

Theorem 1 ([1]). *Let C be a nonempty closed convex subset of a smooth, strictly convex, and reflexive real Banach space X, $T\colon C \to C$ a nonspreading mapping, and $\{\alpha_n\}$ a sequence of $[0,1)$ such that $\sum_{n=1}^{\infty}(1-\alpha_n) = \infty$. Let $\{x_n\}$ and $\{z_n\}$ be sequences of C defined by $x_1 \in C$ and*

$$\begin{cases} x_{n+1} = \Pi_C J^{-1}\left(\alpha_n Jx_n + (1-\alpha_n)JTx_n\right), \\ z_n = \dfrac{1}{\sum_{k=1}^{n}(1-\alpha_k)}\Big((1-\alpha_1)Tx_1 + (1-\alpha_2)Tx_2 + \cdots + (1-\alpha_n)Tx_n\Big) \end{cases}$$

for all $n \in \mathbb{N}$, where Π_C denotes the generalized projection of X onto C. Then $\lim_{n\to\infty}\|z_n\| = \infty$ if and only if T has no fixed point.

[1] Fumiaki Kohsaka, *Averaged sequences for nonspreading mappings in Banach spaces*, Banach and function spaces IV, 2014, pp. 313–323.

[2] Fumiaki Kohsaka and Wataru Takahashi, *Fixed point theorems for a class of nonlinear mappings related to maximal monotone operators in Banach spaces*, Arch. Math. (Basel) **91** (2008), 166–177.

[3] R. T. Rockafellar, *On the maximality of sums of nonlinear monotone operators*, Trans. Amer. Math. Soc. **149** (1970), 75–88.

Sep–2 1720–1740

Attractive points and fixed point properties for nonlinear mappings

Sachiko Atsushiba

Department of Science Education, Graduate School of Education Science of Teaching and Learning, University of Yamanashi, 4-4-37, Takeda Kofu, Yamanashi 400-8510, Japan
Email **asachiko @yamanashi.ac.jp**

In this talk, we study the concepts of acute points of a nonlinear mapping. Then, we study fixed point properties for nonlinear mappings. We also study some properties of acute points, attractive points and fixed points. Further, we prove some convergence theorems for nonlinear mappings.

Some new partial answers to a 52 year old interpolation question

Michael Cwikel

Department of Mathematics, Technion - Israel Institute of Technology, Haifa, 32000, Israel
Email **mcwikel @ math.technion.ac.il**

It is now more than 52 years since Studia Mathematica received Alberto Calderón's very remarkable paper about his theory of complex interpolation spaces. And one of the questions which Calderón implicitly asked in that paper, by solving it in a significant special case, is apparently still open today: ***Does complex interpolation preserve the compactness of an operator?***

After briefly surveying attempts to solve this question over several decades, I will also report on a few new partial answers obtained during the past year, some of them (`arXiv:1411.0171`) jointly with Richard Rochberg. Among other things there is an interplay with Jaak Peetre's "plus-minus" interpolation method, (`arXiv:1502.00986`) a method which probably deserves to be better known. Banach lattices and UMD spaces also have some roles to play.

Several distinguished mathematicians have expressed the belief that that the general answer to this question will ultimately turn out to be negative. Among other things, I will try to hint at where a counterexample might perhaps be hiding. You are all warmly invited to seek it out, or prove that it does not exist.

You can consult `arXiv:1410.4527` for a fairly recent survey which discusses this question.

Sep–3 1020–1100

Linear Extensions and their Applications

David Yost

Centre for Informatics & Applied Optimization,
Federation University,
PO Box 663, Ballarat, Vic. 3353, Australia
Email **d.yost @federation.edu.au**

The following rather general problem occurs in many situations: When can a mapping defined on a topological space space be extended to a larger domain space? Of course the original mapping should have some property (e.g. being continuous, Lipschitz or linear) related to the structure of its domain and range, and it is anticipated that the extension will have the same property. This talk will survey some major results in this area, including the reformulation of selection and embedding problems. Special emphasis will be given to the case of linear mappings between Banach spaces and the fundamental role played by intersecting balls.

Sep–3 1120–1150

Local Kadec-Klee properties in symmetric function spaces

Ryszard Płuciennik

Institute of Mathematics, Poznan University of Technology,
Poznan, Poland
Email **ryszard.pluciennik @ put.poznan.pl**

The essential question in the local geometry of Banach symmetric spaces is whether a point x has some local property P if and only its nonincreasing rearrangement x^* has the same property P. The positive answer is very useful in verifying local properties. The goal of the talk is to present the structure of H_g and H_l-points from that point of view. As an application of these results, several results concerning local approach to Kadec-Klee properties with respect to global (local) convergence in measure in symmetric Banach function spaces will be presented. Some of results are a generalization of the characterization of global Kadec-Klee properties from [1]. Moreover, we notice that, for an H_g-point, the norm is lower simicontinuous with respect to the global convergence in measure, similarly as, for the point of order continuity, the norm is lower simicontinuous with respect to the convergence a.e. A full characterization of H_g and H_l-points in the spaces $L^1 + L^\infty$, $L^1 \cap L^\infty$, Lorentz spaces $\Gamma_{p,w}$ and $\Lambda_{p,w}$ will be given as a consequences of our general results.

(All these results are obtained jointly with Maciej Ciesielski and Paweł Kolwicz)

[1] V. I. Chilin, P. G. Dodds, A. A. Sedaev, and F. A. Sukochev, *Characterizations of Kadec-Klee properties in symmetric spaces of measurable functions*, Trans. Amer. Math. Soc. 348 (12) (1996) 4895-4918.

Sep–3 1150–1220

Which Banach space has James constant $\sqrt{2}$?

Kichi-Suke Saito*, Naoto Komuro and Ryotaro Tanaka

Institute of Science and Technology, Niigata University, Niigata, 950-2181, Japan
Email **saito @ math.sc.niigata-u.ac.jp**

Department of Mathematics, Hokkaido University of Education, Asahikawa Campus, Asahikawa 070-8621, Japan
Email **komuro @ asa.hokkyodai.ac.jp**

Department of Mathematical Science, Graduate School of Science and Technology, Niigata Universirty, Niigata 950-2181, Japan
Email **ryotarotanaka @ m.sc.niigata-.ac.jp**

In 1990, Gao and Lau [1] introduced the James constant of a Banach space as a measure of how "non-square" the unit ball is. Namely, the James constant $J(X)$ of X is given by

$$J(X) = \sup\{\min\{\|x+y\|, \|x-y\|\} : x, y \in S_X\}.$$

As in [1], $\sqrt{2} \leq J(X) \leq 2$ for any normed linear space X. In particular, if X is an inner product space, then $J(X) = \sqrt{2}$. However, the converse does not hold in general.

The aim of this paper is to study Banach spaces with James constant $\sqrt{2}$.

[1] J. Gao and K.-S. Lau, *On the geometry of spheres in normed linear spaces*, J. Aust. Math. Soc. Ser. A, **48** (1990), 101–112.

Sep–3 1400–1420

The Stechkin Inequality for Fourier Multipliers on Nakano Spaces

Alexei Yu. Karlovich

Departamento de Matemática, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Quinta da Torre, 2829-516 Caparica, Portugal
Email **oyk @ fct.unl.pt**

Let $p : \mathbb{R} \to [1, \infty]$ be a measurable a.e. finite function. The Nakano space $L^{p(\cdot)}(\mathbb{R})$ is the collection of all measurable complex-valued functions functions f on $\mathbb{R}$ such that $\int_{\mathbb{R}} |f(x)/\lambda|^{p(x)} dx < \infty$ for some $\lambda = \lambda(f) > 0$. It is well known that it is a Banach function space (in the sense of Luxembrurg) when equipped with the norm

$$\|f\|_{p(\cdot)} = \inf \left\{ \lambda > 0 : \int_{\mathbb{R}} |f(x)/\lambda|^{p(x)} dx \le 1 \right\}.$$

We denote by $\mathcal{B}_M(\mathbb{R})$ the set of all measurable functions $p : \mathbb{R} \to [1, \infty]$ such that

$$1 < \operatorname*{ess\,inf}_{x \in \mathbb{R}} p(x), \quad \operatorname*{ess\,sup}_{x \in \mathbb{R}} p(x) < \infty,$$

and the Hardy-Littlewood maximal operator is bounded on $L^{p(\cdot)}(\mathbb{R})$. Let F and F^{-1} denote the direct and inverse Fourier transform, respectively. A function $a \in L^\infty(\mathbb{R})$ is called a Fourier multiplier on $L^{p(\cdot)}(\mathbb{R})$ if the map

$$f \mapsto F^{-1} a F f$$

maps $L^2(\mathbb{R}) \cap L^{p(\cdot)}(\mathbb{R})$ into itself and extends to a bounded operator on $L^{p(\cdot)}(\mathbb{R})$. The latter convolution operator is then denoted by $W^0(a)$. Our main result is the following generalization of the Stechkin inequality for Fourier multipliers.

Theorem 1. *Let $p \in \mathcal{B}_M(\mathbb{R})$. If a has a finite total variation $V(a)$, then the convolution operator $W^0(a)$ is bounded on the variable Lebesgue space $L^{p(\cdot)}(\mathbb{R})$ and*

$$\|W^0(a)\|_{\mathcal{L}(L^{p(\cdot)})} \le \|H\|_{\mathcal{L}(L^{p(\cdot)})} (\|a\|_\infty + V(a)),$$

where H is the Hilbert transform.

On A-direct sums of Banach spaces

Takayuki Tamura*, Sompong Dhompongsa and Mikio Kato

Graduate School of Humanities and Social Sciences, Chiba University
Chiba 263-8522, Japan
Email **tamura @le.chiba-u.ac.jp**

Department of Mathematics, Faculty of Science, Chiang Mai University,
Chiang Mai 50200, Thailand
Email **sompong.d @cmu.ac.th.**

Kyushu Institute of Technology, Kitakyushu 804-8550, Japan
Email **katom @mns.kyutech.ac.jp**

In the Banach space geometry Z-direct sums and ψ-direct sums of N Banach spaces are often treated, which are defined respectively by means of a so-called "Z-norm" and "ψ-norm" on $\mathbb{R}^N$, ψ is in a class of convex functions Ψ_N. (A ψ-norm is a Z-norm.) More generally we shall discuss *A-direct sums* which are constructed from an *arbitrary* norm $\|\cdot\|_A$ on $\mathbb{R}^N$.

We shall first see any A-direct sum is isometrically isomorphic to a ψ-direct sum with some $\psi \in \Psi_N$. In particular, a Z-direct sum is isometrically isomorphic to a ψ-direct sum. Also we shall discuss some recent results on the uniform non-squareness for A-direct sums of Banach spaces with a strictly monotone norm.

[1] S. Dhompongsa, A. Kaewcharoen and A. Kaewkhao, *Fixed point property of direct sums*, Nonlinear Anal. **63** (2005), e2177-e2188.

[2] P. N. Dowling and S. Saejung, *Non-squareness and uniform non-squareness of Z-direct sums*, J. Math. Anal. Appl. **369** (2010), 53–59.

[3] J. García-Falset, E. Llorens-Fuster and E. M. Mazcuñan-Navarro, *Uniformly nonsquare Banach spaces have the fixed point property for nonexpansive mappings*, J. Funct. Anal. **233** (2006), 494–514.

[4] M. Kato, K.-S. Saito and T. Tamura, *On the ψ-direct sums of Banach spaces and convexity*, J. Aust. Math. Soc. **75** (2003), 413–422.

[5] M. Kato and T. Tamura, Weak nearly uniform smoothness of ψ-direct sums $(X_1 \oplus \cdots \oplus X_N)_\psi$, Comment. Math. **52** (2012), 171–198.

[6] M. Kato and T. Tamura, *Direct sums of Banach spaces with FPP which fail to be uniformly non-square*, J. Nonlinear Convex Anal., **16** (2015), 231-241.

Sep–3 1440–1500

M-constants in some Banach lattices

Paweł Foralewski

Faculty of Mathematics and Computer Science, Adam Mickiewicz University in Poznań,
Umultowska 87, 61-614 Poznań, Poland
Email **katon @amu.edu.pl**

In this talk we will recall some facts concerning M-constants in Banach lattices and we will present some new results concerning these constants in Orlicz-Lorentz spaces.

Sep–3 1520–1540

On the von Neumann-Jordan constant for Banaś-Frączek space

Ken-Ichi Mitani*, Yasuji Takahashi and Kichi-Suke Saito

Department of Systems Engineering, Okayama Prefectural University, Soja 719-1197, Japan
Email **mitani @cse.oka-pu.ac.jp**

Department of Systems Engineering, Okayama Prefectural University, Soja 719-1197, Japan
Email **ym-takahashi @clear.ocn.ne.jp**

Department of Mathematics, Faculty of Science, Niigata University, Niigata 950-2181, Japan
Email **saito @math.sc.niigata-u.ac.jp**

In this talk, we present some recent results on the von Neumann-Jordan constant $C_{\mathrm{NJ}}(X)$ of concrete Banach spaces X. In particular, we treat Banaś-Frączek space $\mathbb{R}^2_\lambda$ introduced in [1], which may be considered as a generalization of Day-James ℓ_2-ℓ_1 space. Recently, the constant $C_{\mathrm{NJ}}(\mathbb{R}^2_\lambda)$ was calculated by Yang [3]. We shall give its simple proof by using Banach-Mazur distance $d(X,Y)$ for $X = \mathbb{R}^2_\lambda$ and some suitable inner product space Y. Moreover, we consider more general two-dimensional normed spaces X and calculate the constant $C_{\mathrm{NJ}}(X)$.

[1] J. Banaś, K. Frączek, Deformation of Banach spaces, Comment. Math. Univ. Carolin. **34** (1993), 47–53.

[2] M. Kato, L. Maligranda, Y. Takahashi, On James and Jordan-von Neumann constants and the normal structure coefficient of Banach spaces, Studia Math. **144** (2001), 275–295.

[3] C. Yang, Jordan-von Neumann constant for Banaś-Frączek space, Banach J. Math. Anal. **8** (2014), 185–192.

[4] C. Yang, F. Wang, On a new geometric constant related to the von Neumann-Jordan constant, J. Math. Anal. Appl. **324** (2006), 555–565.

Sep–3 1540–1600

The duality equality for James constant of Banach spaces

Ryotaro Tanaka* and Kichi-Suke Saito

Department of Mathematical Science,
Graduate School of Science and Technology,
Niigata University, Niigata 950-2181, Japan
Email **ryotarotanaka @m.sc.niigata-u.ac.jp**

Department of Mathematics, Faculty of Science,
Niigata University, Niigata 950-2181 Japan
Email **saito @math.sc.niigata-u.ac.jp**

For a Banach space X, let S_X be the unit sphere of X. The James constant $J(X)$ of a Banach space X was defined in 1990 by Gao and Lau as a measure of how "non-square" the unit ball is, namely, it is given by $J(X) = \sup\{\min\{\|x+y\|, \|x-y\|\} : x, y \in S_X\}$. As with von Neumann-Jordan constant, James constant is known as one of the most important geometric constant of Banach spaces. However, unlike von Neumann-Jordan constant, James constant does not satisfy the duality equality in general. There exists a specific example of two-dimensional space X such that $J(X^*) \neq J(X)$. Hence it is natural to ask when does the duality equality hold for James constant.

In this talk, we consider the above problem for two-dimensional normed spaces. A simple sufficient condition is given. This talk is a joint work with Masahiro Sato who was a graduate student of Niigata University.

Fixed Point and Weak Convergence Theorems for Nonlinear Hybrid Mappings in Banach Spaces

Mayumi Hojo* and Wataru Takahashi

Center for promotion of Educational Innovation, Shibaura Institute of Technology, Fukasaku, Minuma-ku, Saitama, 337-8570, Japan
Email **mayumi-h @ shibaura-it.ac.jp**

Center for Fundamental Science, Kaohsiung Medical University, Kaohsiung 80702, Taiwan; Keio Research and Education Center for Natural Sciences, Keio University, Kouhoku-ku, Yokohama 223-8521, Japan; and Department of Mathematical and Computing Sciences, Tokyo Institute of Technology, Ookayama, Meguro-ku, Tokyo 152-8552, Japan
Email **wataru @ is.titech.ac.jp; wataru@a00.itscom.net**

Hsu, Takahashi and Yao [1] defined a class of nonlinear mappings in a Banach space containing nonexpansive mappings, nonspreading mappings and hybrid mappings as follows: Let E be a Banach space and let C be a nonempty subset of E. A mapping $T : C \to E$ is called generalized hybrid if there are $\alpha, \beta \in \mathbb{R}$ such that

$$\alpha\|Tx - Ty\|^2 + (1-\alpha)\|x - Ty\|^2 \leq \beta\|Tx - y\|^2 + (1-\beta)\|x - y\|^2 \quad (1)$$

for all $x, y \in C$. They called such a mapping (α, β)-generalized hybrid.

We consider an extension of generalized hybrid mappings in a Banach space: A mapping $T : C \to E$ is called extended generalized hybrid if there are $\alpha, \beta, \gamma, \delta \in \mathbb{R}$ such that $\alpha + \beta + \gamma + \delta \geq 0$, $\alpha + \beta > 0$ and

$$\alpha\|Tx - Ty\|^2 + \beta\|x - Ty\|^2 + \gamma\|Tx - y\|^2 + \delta\|x - y\|^2 \leq 0 \quad (2)$$

for all $x, y \in C$. We call such a mapping $(\alpha, \beta, \gamma, \delta)$-extended generalized hybrid.

In this talk, we first obtain some properties for extended generalized hybrid mappings in a Banach space. Then, we prove fixed point and weak convergence theorems of Mann's type for such mappings in a Banach space satisfying Opial's condition.

[1] M.-H. Hsu, W. Takahashi and J.-C. Yao, *Generalized hybrid mappings in Hilbert spaces and Banach spaces*, Taiwanese J. Math. **16** (2012), 129–149.

Topology on ν-generalized metric spaces

Tomonari Suzuki

Department of Basic Sciences, Faculty of Engineering,
Kyushu Institute of Technology, Tobata, Kitakyushu 804-8550, Japan
Email **suzuki-t @ mns.kyutech.ac.jp**

In 2000, Branciari gave the following interesting concept. In our talk, we discuss topologies on ν-generalized metric spaces.

Definition 1 (Branciari [2]). Let X be a set, let d be a function from $X \times X$ into $[0, \infty)$ and let $\nu \in \mathbb{N}$. Then (X, d) is said to be a *ν-generalized metric space* if the following hold:

(N1) $d(x, y) = 0$ iff $x = y$ for any $x, y \in X$.

(N2) $d(x, y) = d(y, x)$ for any $x, y \in X$.

(N3) $d(x, y) \leq d(x, u_1) + d(u_1, u_2) + \cdots + d(u_{\nu-1}, u_\nu) + d(u_\nu, y)$ for any $x, u_1, u_2, \cdots, u_\nu, y \in X$ such that $x, u_1, u_2, \cdots, u_\nu, y$ are all different.

[1] B. Alamri, T. Suzuki and L. A. Khan, *Caristi's fixed point theorem and Subrahmanyam's fixed point theorem in ν-generalized metric spaces*, J. Funct. Spaces, 2015, Art. ID 709391, 6 pp.

[2] A. Branciari, *A fixed point theorem of Banach-Caccioppoli type on a class of generalized metric spaces*, Publ. Math. Debrecen, 57 (2000), 31–37. MR1771669

[3] T. Suzuki, *Generalized metric spaces do not have the compatible topology*, Abstr. Appl. Anal., 2014, Art. ID 458098, 5 pp.

[4] T. Suzuki, B. Alamri and L. A. Khan, *Some notes on fixed point theorems in ν-generalized metric spaces*, Bull. Kyushu Inst. Technol., 62 (2015), 15–23.

[5] T. Suzuki, B. Alamri and M. Kikkawa, *Only 3-generalized metric spaces have a compatible symmetric topology*, submitted.

[6] ______, *Edelstein's fixed point theorem in generalized metric spaces*, submitted.

Sep–3 1700–1720

Fourier transformation of L^2_{loc}-functions

Yoshifumi Ito

209-15 Kamifukuma Hachiman-cho
Tokushima 770-8073, Japan
Email **yoshifumi @md.pikara.ne.jp**

In this paper, we prove Paley-Wiener type theorem for L^2-functions and the structure theorem for the Fourier images of L^2_c and L^2_{loc}.

Theorem 1(Paley-Wiener type theorem)@ *A function $F(\zeta)$ is equal to the Fourier-Laplace transformation*

$$F(\zeta) = \frac{1}{(\sqrt{2\pi})^d} \int f(x) e^{-i\zeta x} dx, \ (\zeta \in \mathbf{C}^d)$$

of a certain L^2-function f such that the condition $\mathrm{supp}(f) \subset \{|x| \leq B\}$ holds for some positive constant B if and only if the function $F(\zeta)$ is an entire function on $\mathbf{C}^d$ such that it satisfies the inequality

$$|F(\zeta)| \leq C e^{B|\mathrm{Im}\,\zeta|}, \ (\zeta \in \mathbf{C}^d)$$

for a certain positive constant C and $F(\xi)$ belongs to $L^2 = L^2(\mathbf{R}^d)$.

Let $\{K_j\}$ be the exhausting sequence of compact sets in $\mathbf{R}^d$.

Theorem 2@*Assume that L^2_c is the TVS of all L^2-functions with compact support in $\mathbf{R}^d$. Then we have the following isomorphisms $(1) \sim (4)$:*

(1)@$L^2_c \cong \varinjlim L^2(K_j) \cong \bigcup_{j=1}^{\infty} L^2(K_j)$. (2)@$\mathcal{F}L^2_c \cong \varinjlim \mathcal{F}L^2(K_j)$.

(3)@$\mathcal{F}L^2(K_j) \cong L^2(K_j), \ (j = 1, \ 2, \ 3, \ \cdots)$.

(4)@$\mathcal{F}L^2_c \cong L^2_c, \ \mathcal{F}L^2_c \subset L^2, \ L^2_c \subset L^2$.

Theorem 3@*We use the usual notation. Then we have the following isomorphisms $(1) \sim (3)$ and the relation (4):*

(1)@$L^2_{\mathrm{loc}} \cong \varprojlim L^2(K_j) \cong \bigcap_{j=1}^{\infty} L^2(K_j) \cong (L^2_c)'$. (2) @$\mathcal{F}L^2_{\mathrm{loc}} \cong \varprojlim \mathcal{F}L^2(K_j)$.

(3)@$\mathcal{F}L^2_{\mathrm{loc}} \cong L^2_{\mathrm{loc}}$. (4)@$\mathcal{F}L^2_{\mathrm{loc}} \subset \mathcal{F}\mathcal{D}', \ \mathcal{F}L^2_{\mathrm{loc}} \neq L^2_{\mathrm{loc}}, \ L^2_{\mathrm{loc}} \subset \mathcal{D}'$.

Reference

[1] Y. Ito, *Fourier Analysis*, preprint, 2015.

Sep–3 1720–1740

Separativity of Ultradifferentiable classes

Waichiro Matsumoto

Department of Applied Mathematics and Informatics,
Ryukoku University,
Seta, Otsu 520-2194, Japan
Email **waichiro @math.ryukoku.ac.jp**

We sometimes consider the Cauchy problems to partial differential equations in a ultradifferntiable class. Let $\{M_n\}_{n=0}^{\infty}$ be a logarithmically convex sequence of positive numbers satisfying $\lim_{n\to\infty}(M_n)^{1/n} = \infty$, $\mathbf{Z}_+$ be the set of the nonnegative integers , K be a compact set in $\mathbf{R}^\ell$ ($\ell \geq 2$), α be a multi index $(\alpha_1, \alpha_2, \ldots, \alpha_\ell)$ in ${\mathbf{Z}_+}^\ell$, $|\alpha|$ be $\alpha_1 + \alpha_2 + \cdots + \alpha_\ell$. We set

$$C\{M_n\}_R(K) = \{f(x) \in C^\infty(K) :\ \exists C > 0,\ |f^{(\alpha)}(x)| \leq CR^{|\alpha|}M_{|\alpha|}\}\,.$$

When $M_n = n!$, this is the space of the analytic functions with radius of convergence $1/R$. For $M_n = n!^\nu$ ($\nu > 1$), this is called Gevray class.

In case of $K = K_1 \times K_2$, K_i a compact in $\mathbf{R}^{\ell_i}$ ($\ell_1 + \ell_2 = \ell$), $\alpha = (\alpha_1, \alpha_2)$ ($\alpha_i \in {\mathbf{Z}_+}^{\ell_i}$), we also set

$$C\{M_n\}_R(K_1\times K_2) = \{f(x) \in C^\infty(K) : \exists C > 0, |f^{(\alpha)}(x)| \leq CR^{|\alpha|}M_{|\alpha_1|}M_{|\alpha_2|}\}\,.$$

We say $C\{M_n\}_R(K)$ is *separative* if $C\{M_n\}_R(K) \subset C\{M_n\}_{R'}(K_1\times K_2)$ for a suitable R'. If all $f(x)$ were separated in the form $f(x_1)f(x_2)$ ($x_i \in \mathbf{R}^{\ell_i}$), this might be true. (Someone call this *ultradifferentiability.*) The separativity of $C\{M_n\}_R(K)$ is brought from the following condition:

$$M_{m+n} \leq R_\circ^{m+n} M_m\, M_n \qquad (\exists R_\circ \geq 1) \tag{1}$$

We characterize Condition (1) and show

Theorem 1. *Condition* (1) $\Rightarrow M_n \leq n!^\nu$ $\quad(\exists\, \nu > 0)$.
The converse is not always true.

The results are announced in [2, 3].

[1] S. Mandelbrojt, Série Adhérentes, Régularization des Suites, Applications, Chap.s I et VI Gauthier-Villars, Paris 1952.

[2] W. Matsumoto, Characterization of the separativity of ultradifferentiable classes, Journal Math Kyoto Univ., vol. 24, no. 4, 1984, 667-678.

[3] ———, Theory of pseudo-differential operators of ultradifferentiable classes, Appendix B, Journal Math Kyoto Univ., vol. 27, no. 3, 1987, 453-500.

Sep–4 0930–1010

Trend constants for Lipschitz mappings

Kazimierz Goebel

Maria Curie-Sklodowska University, Lublin, Poland
Email **goebel @hektor.umcs.lublin.pl**

Lipschtz mappings on convex sets in Banach spaces can be classified in various ways. The standard one is the size of Lipschitz constant, contractions, nonexpansive mappings, mean Lipschtz mappings, asymptotic and uniform nonexpansiveness, etc.. The closer analysis of convex combinations of the mappings with the identity, allows us to introduce some additional constants, called *initial trend* and *terminal trend.* This seems to be a new idea. We present here basic facts and propose some applications to metric fixed point theory.

Sep–4 1020–1100

A universal theorem for stability of almost isometries

Lixin Cheng

School of mathematical Sciences, Xiamen University, Xiamen 361005, China
Email **lxcheng @xmu.edu.cn**

The study of properties of isometries and ε-isometries between Banach spaces has continued for over 80 years.

A mapping f from a Banach space X to another Banach space Y is siad to be an ε-isometry for some $\varepsilon \geq 0$ provided

$$\Big| \|f(x) - f(y)\| \Big| \leq \varepsilon, \text{ for all } x \in X.$$

In this talk, we present a sharp weak stability theorem for a general ε-isometry $f : X \to Y$ with $f(0) = 0$:

For every $x^* \in X^*$ there is $\varphi \in Y^*$ with $\|\varphi\| = \|x^*\| \equiv r$ so that

$$\Big| \langle x^*, x\rangle - \langle \varphi, f(x)\rangle \Big| \leq 2\varepsilon r, \text{ for all } x \in X.$$

It can also be regarded as a universal stability theorem because it unifies, generalizes and improves a series of known results.

Sep–4 1120–1150

Complex interpolation of Morrey spaces–second method

Yoshihiro Sawano* and Denny Ivanal Hakim

Department of Mathematics and Information Science, Tokyo Metropolitan University,
Hachioji 192-0397, Japan
Email **yoshihiro-sawano @celery.ocn.ne.jp**

Department of Mathematics and Information Science, Tokyo Metropolitan University,
Hachioji 192-0397, Japan
Email **dennyivanalhakim @gmail.com**

It is known in [2] that Morrey spaces are closed under the second complex interpolation functor introduced by Calderón. In this talk, we shall discuss the interpolation for the closure in Morrey spaces of the set of all compactly supported functions. We also present the description of the interpolation of these spaces. Our results extend the interpolation theorem for the closure of essentially bounded and compactly supported functions with respect to the Morrey norm, discussed in [1].

[1] D. I. Hakim and Y. Sawano, Interpolation Property of Generalized Morrey spaces, submitted.

[2] P. G. Lemarié-Rieusset, Erratum to: Multipliers and Morrey spaces, 2014.

Sep–4 1150–1220

Carleson inequalities for $L^{(\alpha)}$-harmonic functions

Noriaki Suzuki

Department of Mathematics, Meijo University,
Tenpaku, Nagoya 468-8502, Japan
Email **suzukin @ meijo-u.ac.jp**

We define Hardy and Bergman type Banach spaces with respect to a parabolic operator and discuss Carleson inequalities on those Banach spaces. To be precise, for $n \geq 1$, let $\mathbb{R}^{n+1}_+ := \{(x,t) \in \mathbb{R}^{n+1} \mid x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n, t > 0\}$ denote the upper half space. For $0 < \alpha \leq 1$, let $L^{(\alpha)}$ be a parabolic operator

$$L^{(\alpha)} := \partial_t + (-\Delta_x)^\alpha, \quad \Delta_x := \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \cdots + \frac{\partial^2}{\partial x_n^2}.$$

We say that a continuous function u on $\mathbb{R}^{n+1}_+$ is an $L^{(\alpha)}$-harmonic function if $L^{(\alpha)}u = 0$ in the sense of distributions.

For $1 < p < \infty$, we denote by $h^p_\alpha := h^p_\alpha(\mathbb{R}^{n+1}_+)$ the set of all $L^{(\alpha)}$-harmonic functions u with $\|u\|_{h^p_\alpha} < \infty$, and also for $1 \leq p < \infty$, we denote by $b^p_\alpha := b^p_\alpha(\mathbb{R}^{n+1}_+)$ the set of all $L^{(\alpha)}$-harmonic functions u with $\|u\|_{b^p_\alpha} < \infty$, where

$$\|u\|_{h^p_\alpha} := \sup_{t>0} \left(\int_{\mathbb{R}^n} |u(x,t)|^p dx \right)^{\frac{1}{p}} \text{ and } \|u\|_{b^p_\alpha} := \left(\iint_{\mathbb{R}^{n+1}_+} |u(x,t)|^p dxdt \right)^{\frac{1}{p}}.$$

It is shown that h^p_α and b^p_α are Banach spaces under the norm $\|\cdot\|_{h^p_\alpha}$ and $\|\cdot\|_{b^p_\alpha}$, respectively. We call h^p_α the α-parabolic Hardy space of order p and b^p_α the α-parabolic Bergman space of order p.

The fundamental solutions $W^{(\alpha)}$ of $L^{(\alpha)}$ is given by

$$W^{(\alpha)}(x,t) = \begin{cases} (2\pi)^{-n} \int_{\mathbb{R}^n} e^{-t|\xi|^{2\alpha}} e^{ix\cdot\xi}\, d\xi & t > 0 \\ 0 & t \leq 0 \end{cases}$$

where $x \cdot \xi$ is the inner product of x and ξ. Then

$$|\partial_x^\beta \partial_t^k W^{(\alpha)}(x,t)| \leq C \frac{t^{1-k}}{(t + |x|^{2\alpha})^{\frac{n+|\beta|}{2\alpha}} + 1} \quad (t > 0).$$

When u belongs to h^p_α or b^p_αCwe have

$$u(x,t) = \int_{\mathbb{R}^n} W^{(\alpha)}(x-y, t-s) u(y,s) dy \quad (0 < s < t).$$

It is known that when $\alpha = \frac{1}{2}$, $W^{(\frac{1}{2})}$ coincides with the Poisson kernel on $\mathbb{R}^{n+1}_+$, that is, for $t > 0$,

$$W^{(\frac{1}{2})}(x,t) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\pi^{\frac{n+1}{2}}} \frac{t}{(|x|^2+t^2)^{\frac{n+1}{2}}}.$$

This fact implies that $h^p_{\frac{1}{2}}$ and $b^p_{\frac{1}{2}}$ are the usual harmonic Hardy and Bergman spaces on upper half space, respectively.

Let $1 < p < \infty$ and $0 < q < \infty$. We say that a positive Borel measure μ on $\mathbb{R}^{n+1}_+$ satisfies a (p,q)-Carleson inequality on α-parabolic Hardy spaces if

$$\|u\|_{L^q(\mathbb{R}^{n+1}_+, d\mu)} \le C\|u\|_{h^p_\alpha}$$

holds for all $u \in h^p_\alpha$, and μ satisfies a (p,q)-Carleson inequality on α-parabolic Bergman spaces if

$$\|u\|_{L^q(\mathbb{R}^{n+1}_+, d\mu)} \le C\|u\|_{b^p_\alpha}$$

holds for all $u \in b^p_\alpha$. We will give a necessary and sufficient conditions for a measure μ to satisfy each inequality. As a result, when $1 < p \le q < \infty$ and $1 \le p' \le q' < \infty$, μ satisfies a (p,q)-Carleson inequality on α-parabolic Hardy spaces if and only if μ satisfies a (p',q')-Carleson inequality on α-parabolic Bergman spaces, where $pq'n = p'q(n+2\alpha)$.

[1] H.Nakagawa and N.Suzuki, Carleson inequalities on parabolic Hardy spaces, to appear in Hokkaido Math. J.

[2] M.Nishio, K.Shimomura and N.Suzuki, α-Parabolic Bergman spaces, Osaka J. Math., 42 (2005), 133-162.

[3] M.Nishio, N.Suzuki and M.Yamada, Carleson inequalities on parabolic Bergman spaces, Tohoku Math. J., 62 (2010), 269-286.

[4] ——, Weighted Berezin transformations with appication to Toeplitz operators of Schatten class on parabolic Bergman spaces, Kodai Math. J. 32 (2009), 501-520.

[5] ——, Interporating sequences of parabolic Bergman spaces, Potential Analysis, 28 (2008), 257-278.

[6] ——, Toeplitz operators and Carleson measures on parabolic Bergman spaces, Hokkaido Math. J. 36 (2007), 563-583.

[7] M.Nishio and M.Yamada, Carleson type measures on parabolic Bergman spaces, J. Math. Soc. Japan, 58 (2006), 83-96.

Sep–4 1400–1420

B^u_w-function spaces and their interpolation[1]

Takuya Sobukawa

Global Education Center, Waseda University,
Nishi-Waseda 1-6-1, Shinjuku-ku, Tokyo 169-8050, Japan
Email **sobu @ waseda.jp**

Let $\mathbb{R}^n$ be the n-dimensional Euclidean space. We denote by Q_r the open cube centered at the origin and sidelength $2r$, or the open ball centered at the origin and of radius r.

Definition 1. *Let $w : (0, \infty) \to (0, \infty)$ be a weight function and let $u \in (0, \infty]$. We define function spaces $B^u_w(E) = B^u_w(E)(\mathbb{R}^n)$ and $\dot{B}^u_w = \dot{B}^u_w(E)(\mathbb{R}^n)$ as the sets of all functions $f \in E_Q(\mathbb{R}^n)$ such that $\|f\|_{B^u_w(E)} < \infty$ and $\|f\|_{\dot{B}^u_w(E)} < \infty$, respectively, where*

$$\|f\|_{B^u_w(E)} = \|w(r)\|f\|_{E(Q_r)}\|_{L^u([1,\infty),dr/r)},$$
$$\|f\|_{\dot{B}^u_w(E)} = \|w(r)\|f\|_{E(Q_r)}\|_{L^u((0,\infty),dr/r)}.$$

In the above we abbreviated $\|f|_{Q_r}\|_{E(Q_r)}$ to $\|f\|_{E(Q_r)}$.

If $E = L^p$, then $\dot{B}^u_w(L^p)(\mathbb{R}^n)$ is the local Morrey-type space introduced by Burenkov and Guliyev (Studia Math. vol. 163, 2004).

If $w(r) = r^{-\sigma}$, $\sigma \geq 0$ and $u = \infty$, we denote $B^u_w(E)(\mathbb{R}^n)$ and $\dot{B}^u_w(E)(\mathbb{R}^n)$ by $B_\sigma(E)(\mathbb{R}^n)$ and $\dot{B}_\sigma(E)(\mathbb{R}^n)$, respectively, which were introduced recently by Komori-Furuya, Matsuoka, Nakai and Sawano (Rev. Mat. Complut.vol. 26, 2013).

In this talk, we treat the interpolation property of B^u_w-function spaces.

Theorem 2. *Assume that a family $\{(E(Q_r), \|\cdot\|_{E(Q_r)})\}_{0<r<\infty}$ has the restriction and decomposition properties above. Let $u_0, u_1, u \in (0, \infty]$, $w_0, w_1 \in \mathcal{W}^\infty$, and $w = w_0^{1-\theta} w_1^\theta$. Assume also that, for some positive constant ϵ, $(w_0(r)/w_1(r))r^{-\epsilon}$ is almost increasing, or, $(w_1(r)/w_0(r))r^{-\epsilon}$ is almost increasing. Then*

$$(\dot{B}^{u_0}_{w_0}(E)(\mathbb{R}^n), \dot{B}^{u_1}_{w_1}(E)(\mathbb{R}^n))_{\theta,\, u} = \dot{B}^u_w(E)(\mathbb{R}^n)$$
$$(B^{u_0}_{w_0}(E)(\mathbb{R}^n), B^{u_1}_{w_1}(E)(\mathbb{R}^n))_{\theta,\, u,\, [1,\infty)} = B^u_w(E)(\mathbb{R}^n).$$

As applications of the interpolation property, we also give the boundedness of linear and sublinear operators. It is known that the Hardy-Littlewood maximal operator, fractional maximal operators, singular and fractional integral operators are bounded on B_σ-Morrey-Campanato spaces. Interpolate these function spaces, the we get the boundedness of these operators on $B^u_w(L_{p,\lambda})$, $\dot{B}^u_w(L_{p,\lambda})$, $B^u_w(\mathcal{L}_{p,\lambda})$ and $\dot{B}^u_w(\mathcal{L}_{p,\lambda})$, which are also generalization of the results on the local Morrey-type spaces $LM_{pu,w}(\mathbb{R}^n)$.

[1]The main part of this talk is the joint work with Professor E. Nakai, Ibaraki University.

Sep–4 1420–1440

On $A^p(G)$-algebras, $1 \le p < \infty$ and the Multipliers of $A^p(G)$ for $1 \le p \le 2$

Hang-Chin Lai

Emeritus Chair Professor of Mathematics
National Tsing Hua University, Hsinchu, Taiwan
Email **laihc @mx.nthu.edu.tw**

Let G be an infinite noncompact LCA group with dual group $\widehat{G}$. Denote $A^p(G)$, $1 \le p < \infty$ by

$$\{f \in L^1(G)| \text{ with Fourier transform } \hat{f} \in L^p(\widehat{G}).\}$$

We supply the norm of $A^p(G)$ by

$$\|f\|^p = \max(\|f\|_1, \|\hat{f}\|_p),$$

which is equivalent to the sum norm

$$\|f\|^p = \|f\|_1 + \|\hat{f}\|_p \text{ for } f \in A^p(G).$$

Then $A^p(G)$ is a commutative Banach algebra under convolution product with norm $\|\cdot\|^p$.

Since $f \in A^p(G)$, $\hat{f} \in L^p(\widehat{G}) \cap C_0(\widehat{G})$ is a bounded continuous function vanishing at infinity of $\widehat{G}$, we see that $L^q(\widehat{G}) \supset L^r(\widehat{G})$ if $q < r < \infty$ and if $1 < p \le 2, \frac{1}{p} + \frac{1}{q} = 1$.

It is easily to see that $A^p(G)$ can not include all $C_c(G)$ since $C_c(G) \subset L^1(G)$, the Fourier transforms of $C_c(G)$ may not be in $L^p(\widehat{G})$. Thus we consider to split $A^p(G)$ to be $1 \le p \le 2$ and $2 \le q < \infty$, one sees that for $f \in A^p(G)$, $1 \le p \le 2$ and $g \in C_c(G) \subset L^1 \cap L^2(G)$, we have $\hat{f} \in L^p(\widehat{G}) \cap C_0(\widehat{G}) \subset L^2(\widehat{G})$ and $\hat{g} \in L^2(\widehat{G}) \cap C_0(\widehat{G})$ such that

$$f * g \in C_0(G),$$

the parseval's identity is applicable that

$$\int_{\widehat{G}} \widehat{f * g} d\hat{x} = \int_G f(x)\tilde{g}(x)dx = f * g(0),$$

where $\tilde{g}(x) = g(-x)$. By this technic, we can constitute the space $A_p(G)$ for $1 < p \le 2$ to be the set of all functions $u(x)$ such that

$$u = \sum_{i=1}^{\infty} f_i * g_i : f_i \in A^p(G), g_i \in C_q = \{g \in C_c \text{ with } \hat{g} \in L^q(\widehat{G})\}$$

and

$$\sum_{i=1}^{\infty} \|f_i\|^p \|g_i\| < \infty, \text{ where } \|g_i\| = \inf(\|g\|_\infty + \|\hat{g}\|_q).$$

Define

$$u \longmapsto \|u\|_p = \inf_u \{\sum_{i=1}^{\infty} \|f_i\|^p \|g_i\|, u = \sum_{i=1}^{\infty} f_i * g_i \text{ in } A_p(G)\}.$$

This space $A_p(G)$ is a dense subspace of $C_0(G)$. Finally, we prove that the multiplier space $\mathcal{M}(A^p)$ for $1 < p \leq 2$ is isometrically isomorphic to the topological dual $A_p(G)^*$ of $A_p(G)$. And by a corollary, it can be shown that $\mathcal{M}(A^1(G)) \cong A_1(G)^*$. Consequently, $\mathcal{M}(A^p) \cong A_p(G)^*$ for $1 \leq p \leq 2$.

Sep–4 1440–1500

A projection method for approximating fixed points of quasinonexpansive mappings in complete metric spaces

Shuechin Huang* and Yasunori Kimura

Department of Applied Mathematics, National Dong Hwa University,
Hualien 97401, Taiwan
Email **shuang @ mail.ndhu.edu.tw**

Department of Information Science, Toho University,
Miyama, Funabashi, Chiba 274-8510, Japan
Email **yasunori @ is.sci.toho-u.ac.jp**

The purpose of this talk is to analyze the feasibility study of Moudafi viscosity type of projection method with a weak contraction for a finite family of quasinonexpansive mappings in a complete CAT(0) space.

Sep–4 1520–1540

Some new convergence theorems for new nonlinear cyclic mappings on quasiordered metric spaces

Ing-Jer Lin* and Yan-Wei Wu

Department of Mathematics, National Kaohsiung Normal University,
Kaohsiung 82444, Taiwan
Email **ijlin @ mail.nknu.edu.tw**

Department of Mathematics, National Kaohsiung Normal University,
Kaohsiung 82444, Taiwan
Email **s0909456 @ gmail.com**

In this paper, we first establish some convergence theorems for the best proximity points on quasiordered metric spaces. We also introduce so-called cyclic weak light deliver mappings and then prove some new convergence theorems for such mappings on usual metric spaces.

An iterative scheme extending Halpern type in a Hadamard space

Hideyuki Wada* and Yasunori Kimura

Department of Information Science, Toho University,
2-2-1, Miyama, Funabashi, Chiba 274-8510, Japan
Email **6514012w @nc.toho-u.ac.jp**

Department of Information Science, Toho University,
2-2-1, Miyama, Funabashi, Chiba 274-8510, Japan
Email **yasunori @is.sci.toho-u.ac.jp**

In this talk, we consider an approximation theorem of fixed points for nonexpansive mappings in a Hadamard space. In 1992, Wittmann [5] shows a Halpern type iteration with a nonexpansive mapping converges strongly to a fixed point in a Hilbert space. Shioji and Takahashi [4] proved the same iteration also converges in the setting of Banach spaces with certain assumptions, and Kimura, Takahashi and Toyoda [1] introduced an iterative scheme extending Halpern type by using a convex combination of mappings.

Saejung [3] introduced a Halpern type iteration with a nonexpansive mapping approximating a fixed point in a Hadamard space. Moreover, it was also showed that an iterative scheme using a convex combination of Halpern type construction is strong convergence to a common fixed point in a Hadamard space [2]. We analyze the coefficient of the iterative sequence more deeply and obtain some convergence results. From this result, we know that the iteration proposed in [2] is a natural extension of Halpern type.

[1] Y. Kimura, W. Takahashi and M. Toyoda, *Convergence to common fixed points of a finite family of nonexpansive mappings*, Arch. Math. (Basel) **84** (2005), 350–363.

[2] Y. Kimura and H. Wada, *Halpern type iteration with multiple anchor points in a Hadamard space*, J. Inequal. Appl. **2015**:182 (2015), 11pp.

[3] S. Saejung, *Halpern's Iteration in CAT(0) Spaces*, Fixed Point Theory and Appl. **2010** (2010), 13pp.

[4] N. Shioji and W. Takahashi, *Strong convergence of approximated sequences for nonexpansive mappings in Banach spaces*, Proc. Amer. Math. Soc. **125** (1997), no. 12, 3641–3645.

[5] R. Wittmann, *Approximation of fixed points of nonexpansive mappings*, Arch. Math. (Basel) **58** (1992), 486–491.

The Mann Algorithm in a Complete Geodesic Space with Curvature Bounded Above

Pongsakorn Yotkaew*, Yasunori Kimura and Satit Saejung

Department of Mathematics, Faculty of Science, Khon Kaen University,
Khon Kaen 40002, Thailand
Email **pongyo @kku.ac.th**

Department of Information Science, Toho University, Miyama,
Funabashi, Chiba 274-8510, Japan
Email **yasunori @is.sci.toho-u.ac.jp**

Department of Mathematics, Faculty of Science, Khon Kaen University,
Khon Kaen 40002, Thailand
Email **saejung @kku.ac.th**

The purpose of this paper is to prove two Δ-convergence theorems of the Mann algorithm to a common fixed point for a countable family of mappings in the case of a complete geodesic space with curvature bounded above by a positive number. The first one for nonexpansive mappings improves the recent result of He *et al.* [1]. The last one is proved for quasi-nonexpansive mappings and applied to the problem of finding a common fixed point of a countable family of quasi-nonexpansive mappings.

[1] He, JS, Fang, DH, López, G, Li, C: Mann's algorithm for nonexpansive mappings in CAT(κ) spaces. Nonlinear Anal. **75**, 445–452 (2012)

Sep–4 1620–1640

Fixed point theorems for widely more generalized hybrid mappings in a Banach space

Toshiharu Kawasaki

College of Engineering, Nihon University,
Koriyama-shi, Fukushima 963-8642, Japan
Email **toshiharu.kawasaki @ nifty.ne.jp**

Let E be a Banach space and let C be a non-empty subset of E. A mapping T from C into E is said to be widely more generalized hybrid if there exist real numbers $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta$ and η such that

$$\begin{aligned}\alpha\|Tx - Ty\|^2 + \beta\|x - Ty\|^2 + \gamma\|Tx - y\|^2 + \delta\|x - y\|^2 \\ +\varepsilon\|x - Tx\|^2 + \zeta\|y - Ty\|^2 + \eta\|(x - Tx) - (y - Ty)\|^2 \leq 0\end{aligned}$$

for any $x, y \in C$. Such a mapping is called an $(\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta)$-widely more generalized hybrid mapping. The definition above introduced by Kawasaki and Takahashi in [5] in the case where E is a Hilbert space. In this talk we also use this definition in the case of Banach space and we show a fixed point theorem in a Banach space and a new fixed point theorem in a Hilbert space.

[1] T. Kawasaki, An extension of existence and mean approximation of fixed points of generalized hybrid non-self mappings in Hilbert spaces, Journal of Nonlinear and Convex Analysis, to appear.

[2] T. Kawasaki, Fixed points theorems and mean convergence theorems for generalized hybrid self mappings and non-self mappings in Hilbert spaces, Pacific Journal of Optimization, to appear.

[3] T. Kawasaki, Fixed point theorem for widely more generalized hybrid demicontinuous mappings in Hilbert spaces, Proceedings of Nonlinear Analysis and Convex Analysis 2015, Yokohama Publishers, Yokohama, to appear.

[4] T. Kawasaki and T. Kobayashi, Existence and mean approximation of fixed points of generalized hybrid non-self mappings in Hilbert spaces, Scientiae Mathematicae Japonicae 77 (Online Version: e-2014) (2014), 13–26 (Online Version: 29–42).

[5] T. Kawasaki and W. Takahashi, Existence and mean approximation of fixed points of generalized hybrid mappings in Hilbert spaces, Journal of Nonlinear and Convex Analysis 14 (2013), 71–87.

[6] T. Kawasaki and W. Takahashi, Fixed point and nonlinear ergodic theorems for widely more generalized hybrid mappings in Hilbert spaces and applications, Proceedings of Nonlinear Analysis and Convex Analysis 2013, Yokohama Publishers, Yokohama, to appear.

[7] T. Kawasaki and W. Takahashi, Fixed point theorems for generalized hybrid demicontinuous mappings in Hilbert spaces, Linear and Nonlinear Analysis 1 (2015), 125–138.

[8] W. Takahashi, Unique fixed point theorems for nonlinear mappings in Hilbert spaces, Journal of Nonlinear and Convex Analysis 15 (2014), 831–849.

Sep–4 1640–1700

On Ishikawa's convergence theorem for pseudo-contractions

Yukio Takeuchi

Takahashi Institute for Nonlinear Analysis,
1-11-11 Nakazato, Minami,Yokohama 232-0063, Japan
Email **aho31415 @yahoo.co.jp**

In 1967, Browder and Petryshyn [1] initiated the study of fixed points of strictly pseudo-contractions. In 1974, Ishikawa [2] made an impact on this study area. On the other hand, in 2011, Takahashi and Takeuchi [4] introduced the concept of attractive points and apply it to have an extension of the Baillon type ergodic theorem due to Kocourek, Takahashi and Yao [3].

In this talk, motivated by the works as above, we deal with the concept of k–acute points of a mapping T, where $k \in [0, 1]$. Then, we present some properties of k-acute points and relations among k–acute points, attractive points and fixed points. Further, we apply these to have some results connected with Ishikawa's convergence theorem in [2].

This talk is based on Takahashi and Takeuchi [4], and a joint work with Professors Atsushiba, Iemoto, and Kubota.

[1] F.E. Browder and W.V. Petryshyn, *Construction of fixed points of nonlinear mappings in Hilbert space*, J. Math. Anal. Appl. **20** (1967) 197–228.

[2] S. Ishikawa, *Fixed points by a new iteration method*, Proc. Amer. Math. Soc. **44** (1974), no.1, 147–150.

[3] P. Kocourek, W. Takahashi and J.-C. Yao, *Fixed point theorems and weak convergence theorems for generalized hybrid mappings in Hilbert spaces*, Taiwanese J. Math. **14** (2010), 2497–2511.

[4] W. Takahashi and Y. Takeuchi, *Nonlinear ergodic theorem without convexity for generalized hybrid mappings in a Hilbert space*, J. Nonlinear Convex Anal. **12** (2011), 399–406.

Sep–5 0930–1010

Mixed-norm Lebesgue spaces: Inequalities and inclusions

Gord Sinnamon* and Wayne Grey

Department of Mathematics, Western University,
London, N6A 5B7 Canada
Email **sinnamon @ uwo.ca**
Email **wgrey @ uwo.ca**

Mixed-norm Lebesgue spaces were introduced in [1] by the formula,

$$\|f\|_P = \left(\int \cdots \left(\int \left(\int |f(x_1, x_2, \ldots, x_n)|^{p_1} \, d\mu_1 \right)^{p_2/p_1} d\mu_2 \right)^{p_3/p_2} \cdots \, d\mu_n \right)^{1/p_n}$$

where $P = (p_1, p_2, \ldots, p_n)$. A systematic method for generating mixed-norm inequalities is presented in [2]. It replaces a number of lengthy and complicated proofs of known inequalities and establishes a large family of new inequalities of a similar form; those involving *permuted* mixed norms.

The inclusion problem for permuted mixed-norm Lebesgue spaces is studied in [3]: Given two such spaces, **when is one space contained in the other?** If so, **what is the norm of the inclusion map?** The answers to these two questions turn out to be unexpectedly subtle and, in certain cases, surprisingly difficult.

In the two-variable case, both questions are settled for a large range of Lebesgue indices and all σ-finite measures. For the remaining indices, both are settled if at least one measure is atomless or if no measure is purely atomic. The first is settled except when all measures are purely atomic.

Still in the two-variable case, estimates are given when all measures are purely atomic. But an exact answer is to much to expect: The second question is intractable for certain indices— it is equivalent to an optimization problem that includes a known NP-hard problem as a special case.

When no measure is purely atomic, the n-variable case reduces to the two-variable case.

[1] A. Benedek and R. Panzone, *The spaces L^p with mixed norm*, Duke Math. J. 26 (1961), 301–324.

[2] W. Grey, *Mixed-norm estimates and symmetric geometric means*, arXiv:1507.08327.

[3] W. Grey and G. Sinnamon, *The inclusion problem for mixed-norm spaces*, Trans. Amer. Math. Soc., to appear.

Approximation of multivariate periodic Sobolev functions

Thomas Kühn

Mathematisches Institut, Universität Leipzig,
Augustusplatz 10, D-04109 Leipzig, Germany
Email **kuehn @math.uni-leipzig.de**

This talk is concerned with approximation of functions on the d-dimensional torus belonging to isotropic Sobolev spaces $H^s(\mathbb{T}^d)$ or to Sobolev spaces $H^s_{\text{mix}}(\mathbb{T}^d)$ of dominating mixed smoothness $s > 0$, where the error is measured in the L_2-norm. The asymptotic order of the corresponding approximation numbers is well known: With certain constants, depending only on the dimension d and the smoothness s, one has in the isotropic case

$$c_s(d)\, n^{-s/d} \le a_n(I_d : H^s(\mathbb{T}^d) \to L_2(\mathbb{T}^d)) \le C_s(d)\, n^{-s/d}\,,$$

and in the mixed case

$$c_s(d) \left[\frac{(\log n)^{d-1}}{n}\right]^s \le a_n(I_d : H^s_{\text{mix}}(\mathbb{T}^d) \to L_2(\mathbb{T}^d)) \le C_s(d) \left[\frac{(\log n)^{d-1}}{n}\right]^s .$$

In the literature almost nothing is known about the involved constants. However, for high-dimensional numerical problems and also for tractability issues in information-based complexity, information on the constants is essential, especially their dependence on d. I will present some new results on

– the exact asymptotic behavior of the constants as $d \to \infty$,

– rate-optimal two-sided estimates of a_n for large n,

– matching two-sided "preasymptotic" estimates for small n,

Moeover, I will give a general method that allows to derive estimates for L_∞-approximation from L_2-approximation. The proofs rely on combinatorial and volume estimates, an interesting connection to entropy numbers in finite-dimensional ℓ_p-spaces, and operator ideal techniques.
The talk is based on the recent joint papers [1]–[4] with F. Cobos (Madrid), S. Mayer (Bonn), W. Sickel (Jena) and T. Ullrich (Bonn).

[1] F. Cobos, T. Kühn and W. Sickel, *Optimal approximation of multivariate periodic Sobolev functions in the sup-norm*, submitted 2014, arXiv:1505.02636

[2] T. Kühn, W. Sickel and T. Ullrich, *Approximation numbers of Sobolev embeddings – Sharp constants and tractability*, J. Complexity 30 (2014), 95–116.

[3] T. Kühn, W. Sickel and T. Ullrich, *Approximation of mixed order Sobolev functions on the d-torus – Asymptotics, preasymptotics and d-dependence*, Constr. Approx. (Online First 2015), arXiv:1312.6386

[4] S. Mayer, T. Kühn and T. Ullrich, *Counting via entropy: new preasymptotics for the approximation numbers of Sobolev embeddings*, submitted 2015, arXiv:1505.00631

Sep–5 1120–1150

On Browder's convergence theorem and Halpern iteration process for G-nonexpansive mappings in Hilbert spaces endowed with graphs

Suthep Suantai

Department of Mathematics, Faculty of Science, Chiang Mai University
Chiang Mai, Thailand
Email **suthep.s @cmu.ac.th**

In this talk, we prove Browder's convergence theorem for G-nonexpansive mappings in a Hilbert space with a directed graph. Moreover, we also prove strong convergent of the Halpern iteration process to a fixed point of G-nonexpansive mappings in a Hilbert space endowed with a directed graph. Examples illustrating our main results are also given. The main results obtained in this paper extend and generalize many known results in the literature therein.

Sep–5 1150–1220

Generalized Quasi-Metric Adjusted Skew Information and Trace Inequality

Kenjiro Yanagi

Graduate School of Science and Engineering, Yamaguchi University,
Ube 755-8611, Japan
Email **yanagi @yamaguchi-u.ac.jp**

Let $M_n(\mathbb{C})$(resp. $M_{n,sa}(\mathbb{C})$) be the set of all $n \times n$ complex matrices (resp. all $n \times n$ self-adjoint matrices), endowed with the Hilbert-Schmidt scalar product $\langle A, B \rangle = Tr(A^*B)$. Let $M_{n,+}(\mathbb{C})$ be the set of strictly positive elements of $M_n(\mathbb{C})$ and $M_{n,+,1}(\mathbb{C})$ be the set of stricly positive density matrices, that is $M_{n,+,1}(\mathbb{C}) = \{\rho \in M_n(\mathbb{C}) | Tr\rho = 1, \rho > 0\}$. A function $f : (0, +\infty) \to \mathbb{R}$ is said operator monotone if, for any $n \in \mathbb{N}$, and $A, B \in M_{n,+}(\mathbb{C})$ such that $0 \le A \le B$, the inequalities $0 \le f(A) \le f(B)$ hold. An operator monotone function is said symmetric if $f(x) = xf(x^{-1})$ and normalized if $f(1) = 1$.

Definition 1. *$\mathcal{F}_{op}$ is the class of functions $f : (0, +\infty) \to (0, +\infty)$ such that*

(1) $f(1) = 1$,

(2) $tf(t^{-1}) = f(t)$,

(3) f *is operator monotone.*

Example 2. *Examples of elements of $\mathcal{F}_{op}$ are given by the following list*

$$f_{RLD}(x) = \frac{2x}{x+1}, \quad f_{WY}(x) = \left(\frac{\sqrt{x}+1}{2}\right)^2, \quad f_{BKM}(x) = \frac{x-1}{\log x},$$

$$f_{SLD}(x) = \frac{x+1}{2}, \quad f_{WYD}(x) = \alpha(1-\alpha)\frac{(x-1)^2}{(x^\alpha - 1)(x^{1-\alpha} - 1)}, \ \alpha \in (0,1).$$

Remark . *Any $f \in \mathcal{F}_{op}$ satisfies*

$$\frac{2x}{x+1} \le f(x) \le \frac{x+1}{2}, \quad x > 0.$$

For $f \in \mathcal{F}_{op}$ define $f(0) = \lim_{x \to 0} f(x)$. We introduce the sets of regular and non-regular functions

$$\mathcal{F}_{op}^r = \{f \in \mathcal{F}_{op} | f(0) \neq 0\}, \quad \mathcal{F}_{op}^n \{f \in \mathcal{F}_{op} | f(0) = 0\}$$

and notice that trivially $\mathcal{F}_{op} = \mathcal{F}_{op}^r \cup \mathcal{F}_{op}^n$.

Definition 3. *Let $g, f \in \mathcal{F}_{op}^r$ satisfy*

$$g(x) \geq k \frac{(x-1)^2}{f(x)}$$

for some $k > 0$. We define

$$\Delta_g^f(x) = g(x) - k\frac{(x-1)^2}{f(x)} \in \mathcal{F}_{op}$$

In Kubo-Ando theory of matrix means one associates a mean to each operator monotone function $f \in \mathcal{F}_{op}$ by the formula

$$m_f(A, B) = A^{1/2} f(A^{-1/2} B A^{-1/2}) A^{1/2},$$

where $A, B \in M_{n,+}(\mathbb{C})$. Using the notion of matrix means one may define the class of generalized monotone metrics (also said generalized quantum Fisher informtions) for $X, Y \in M_n(\mathbb{C})$ by the following formula

$$\langle X, Y \rangle_f = Tr[X^* \cdot m_f(L_A, R_B)^{-1}(Y)],$$

where $L_A(X) = AX, R_B(X) = XB$. When $A = B = \rho \in M_{n,+,1}(\mathbb{C})$ and $X, Y \in M_{n,sa}(\mathbb{C})$, one has to think of X, Y as tangent vectors to the manifold $M_{n,+,1}(\mathbb{C})$ at the point ρ.

Definition 4. *For $X \in M_n(\mathbb{C})$ and $A, B \in M_{n,+}(\mathbb{C})$, we define the following quantities:*

(1) $I_{A,B}^{g,f}(X) = k\langle (L_A - R_B)X, (L_A - R_B)X \rangle_f$,

(2) $Corr_{A,B}^{g,f}(X, Y) = k\langle (L_A - R_B)X, (L_A - R_B)Y \rangle_f$,

(3) $C_{A,B}^f(X) = Tr[X^* \cdot m_f(L_A, R_B)X]$,

(4) $U_{A,B}^{g,f}(X) = \sqrt{(C_{A,B}^g(X) + C_{A,B}^{\Delta_g^f}(X))(C_{A,B}^g(X) - C_{A,B}^{\Delta_g^f}(X))}$.

The quantity $I_{A,B}^{g,f}(X)$ and $Corr_{A,B}^{g,f}(X, Y)$ are said generalized quasi-metric adjusted skew information and generalized quasi-metric adjusted correlation measure, respectively.

Then we have the following proposition.

Proposition 5. *For $X, Y \in M_n(\mathbb{C})$ and $A, B \in M_{n,+}(\mathbb{C})$, we have the following relations.*

(1) $I_{A,B}^{g,f}(X) = C_{A,B}^g(X) - C_{A,B}^{\Delta_g^f}(X)$,

(2) $J^{g,f}_{A,B}(X) = C^{g}_{A,B}(X) + C^{\Delta^f_g}_{A,B}(X)$,

(3) $U^{g,f}_{A,B}(X) = \sqrt{I^{g,,f}_{A,B}(X) \cdot J^{g,f}_{A,B}(X)}$.

Theorem 6. *For $f \in \mathcal{F}^r_{op}$, it holds*

$$U^{g,f}_{A,B}(X) \cdot U^{g,f}_{A,B}(Y) \geq |Corr^{g,f}_{A,B}(X,Y)|^2,$$

where $X, Y \in M_n(\mathbb{C})$ and $A, B \in M_{n,+}(\mathbb{C})$.

Theorem 7. *For $f \in \mathcal{F}^r_{op}$, if*

$$g(x) + \Delta^f_g(x) \geq \ell f(x)$$

for some $\ell > 0$, then it holds

$$U^{g,f}_{A,B}(X) \cdot U^{g,f}_{A,B}(Y) \geq k\ell |Tr[X^* |L_A - R_B| Y]|^2,$$

where $X, Y \in M_n(\mathbb{C})$, $A, B \in M_{n,+}(\mathbb{C})$ and

$$|L_A - R_B| = \sum_{i=1}^{n} \sum_{j=1}^{n} |\lambda_i - \mu_j| L_{|\phi_i\rangle\langle\phi_i|} R_{|\psi_j\rangle\langle\psi_j|},$$

for spectral decompositions $A = \sum_{i=1}^{n} \lambda_i |\phi_i\rangle\langle\phi_i|, \quad B = \sum_{j=1}^{n} \mu_j |\psi_j\rangle\langle\psi_j|$.

Corollary 8. *Under the same assumption in Theorem 7, we have*

$$\begin{aligned} & k\ell (Tr[X^* |L_A - R_B| X])^2 \\ \leq \; & Tr[X^*(m_g(L_A, R_B) - m_{\Delta^f_g}(L_A, R_B))X] Tr[X^*(m_g(L_A, R_B) + m_{\Delta^f_g}(L_A, R_B))X], \end{aligned}$$

where $X \in M_n(\mathbb{C})$ and $A, B \in M_{n,+}(\mathbb{C})$.

Example 9. *When*

$$g(x) = \frac{x+1}{2}, \quad f(x) = \alpha(1-\alpha)\frac{(x-1)^2}{(x^\alpha - 1)(x^{1-\alpha} - 1)}, \quad c = \frac{f(0)}{2}, \quad d = 2,$$

we have the following trace inequality by putting $X = I$.

$$\alpha(1-\alpha)(Tr[|L_A - R_B| I])^2 \leq \left(\frac{1}{2} Tr[A+B]\right)^2 - \left(\frac{1}{2} Tr[A^\alpha B^{1-\alpha} + A^{1-\alpha} B^\alpha]\right)^2.$$

We have the following trace inequality by combining the Chernoff/Powers-Stomner/Audenaert inequality and the above trace inequality.

Theorem 10. *We have the following:*

$$
\begin{aligned}
& \frac{1}{2}Tr[A+B-|L_A-R_B|I] \le \inf_{0\le\alpha\le 1} Tr[A^{1-\alpha}B^{\alpha}] \\
\le \; & Tr[A^{1/2}B^{1/2}] \le \frac{1}{2}Tr[A^{\alpha}B^{1-\alpha}+A^{1-\alpha}B^{\alpha}] \\
\le \; & \sqrt{\left(\frac{1}{2}Tr[A+B]\right)^2 - \alpha(1-\alpha)(Tr[|L_A-R_B|I])^2}.
\end{aligned}
$$

Remark . *We remark the following:*

(1) *The third inequality means that $Tr[A^{1-\alpha}B^{\alpha}]$ is convex in α.*

(2) *There are no relationship between $Tr[|A-B|]$ and $Tr[|L_A-R_B|I]$*

[1] Audenaert K.M.R., Calsamiglia J., Masancs L.I., Munnoz-Tapia R., Acin A., Bagan E. and Verstraete F., The quantum Chernoff bound, Phys. Rev. Lett., 98(2007), 160501.

[2] Audenaert K.M.R., Nussbaum M., Szkola A. and Verstraete F., Asymptotic error rates in quantum hypothesis testing, Commun. Math. Phys., 279(2008), 251-283.

[3] Powers R.T. and Stormer E., Free states of the canonical anticommutation relations, Commun. Math. Phys.,16(1970), 1-33.

[4] Yanagi K., Furuichi S. and Kuriyama K., Uncertainty relations for generalized metric adjusted skew information and generalized metric correlation measure, J. Uncertainty Anal. Appl., 1(2013), 1-14.

[5] Yanagi K., Non-hermitian extensions of Schrödinger type uncertainty relations, Proceedings of ISITA, 2014, 163-166.

[6] Yanagi K., Generalized metric adjusted skew information and uncertainty relation, Proceedings of ISBFS, 2014, 435-442.

[7] Yanagi K. and Sekikawa K., Non-hermitian extensions of Heisenberg type and Schrödinger type uncertainty relations, preprint, 2015.

Bounded convergence theorems for nonlinear integrals

Jun Kawabe

Devision of Mathematics, Faculty of Engineering, Shinshu University,
4-17-1 Wakasato, Nagano 380-8553, Japan
Email **jkawabe @shinshu-u.ac.jp**

In this talk, we introduce a new notion of the perturbation of functional to formulate a functional form of bounded convergence theorem for nonlinear integrals in nonadditive measure theory. As well as the Lebesgue integral, a nonlinear integral may be viewed as a nonlinear functional $I\colon \mathcal{M}(X) \times \mathcal{F}^+(X) \to [0,\infty]$, where X is a non-empty set, $\mathcal{A}$ is a field of subsets of X, $\mathcal{M}(X)$ is the set of all nonadditive measures on $(X, \mathcal{A})$ and $\mathcal{F}^+(X)$ is the set of all $\mathcal{A}$-measurable non-negative functions on X. So, we will formulate our results for such a functional. The key concept is the perturbation of functional that manages not only the monotonicity of the functional but also the small change of the functional value $I(\mu, f)$ caused by adding a small term to a measure μ and a function f. As its direct consequences, we obtain some bounded convergence theorems for typical nonlinear integrals, which show that the autocontinuity of a nonadditive measure is equivalent to the validity of the bounded convergence theorems for the Choquet, the Šipoš, the Sugeno, and the Shilkret integrals as well as their symmetric and asymmetric extensions. Our results are also applicable to the Lebesgue integral when the nonadditive measure μ is σ-additive.

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[1] G. Choquet, Theory of capacities, Ann. Inst. Fourier (Grenoble) 5 (1953-54) 131–295.

[2] J. Kawabe, The bounded convergence in measure theorem for nonlinear integral functionals, Fuzzy Sets Syst. 271 (2015) 31–42.

[3] D. Ralescu, G. Adams, The fuzzy integral, J. Math. Anal. Appl. 75 (1980) 562–570.

[4] N. Shilkret, Maxitive measure and integration, Indag. Math. 33 (1971) 109–116.

[5] J. Šipoš, Integral with respect to a pre-measure, Math. Slovaca 29 (1979) 141–155.

[6] M. Sugeno, Theory of fuzzy integrals and its applications, Ph.D. Dissertation, Tokyo Institute of Technology, Tokyo, 1974.

Sep–5 1420–1440

Spaces with multiweighted derivatives

Ryskul Oinarov* and Aigerim Kalybay

L.N. Gumilyov Eurasian National University,
Munaytpasov St. 5, Astana 010008, Kazakhstan
Email **o_ryskul @ mail.ru**

KIMEP University,
Abai Ave. 4, Almaty 050010, Kazakhstan
Email **kalybay @ kimep.kz**

Let $u_i(\cdot)$, $i = 1, 2, \dots, n$, together with $u_i^{-1}(\cdot)$, $i = 1, 2, \dots, n$, be positive and locally summable functions on $I = (a, b)$, $-\infty \le a < b \le \infty$. Suppose that $\overline{u} = \{u_1, u_2, ..., u_n\}$. For the function $f : I \to R$ we define the following differential operation:

$$D^0_{\overline{u}} f(t) \equiv f(t),\ D^k_{\overline{u}} f(t) := u_k(t) \frac{d}{dt} D^{k-1}_{\overline{u}} f(t),\ t \in I,\ k = 1, 2, \dots, n,$$

where each derivative is generalized ([1], p. 140).

The operation $D^k_{\overline{u}} f(t)$, $k = 1, 2, \dots, n$, is called multiweighted k order derivative of the function f. Denote by $W^n_{p,\overline{u}} := W^n_{p,\overline{u}}(I)$ the set of all functions f locally absolutely continuous on I, which have multiweighted k order derivatives on I, with the finite semi–norm:

$$\|f\|^{(n)}_{p,\overline{u}} = \|D^n_{\overline{u}} f\|_p,$$

where $\|\cdot\|_p$ is the standard norm of the space $L_p(I)$, $1 \le p \le \infty$.

Under assumption that the functions u_i, $i = 1, 2, \dots, n$, vanish at the end-points of I, we investigate properties of the space $W^n_{p,\overline{u}}$ and behavior of functions f from $W^n_{p,\overline{u}}$ at these end-points.

The important partial case for $W^n_{p,\overline{u}}$ is the case when $u_i(t) = t^{\alpha_i}$, $i = 1, 2, \dots, n$. In this case the space $W^n_{p,\overline{u}}$ is denoted by $W^n_{p,\overline{\alpha}}$, where $\overline{\alpha} = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$.

The talk is devoted to properties of the space $W^n_{p,\overline{u}}(I)$ when $I = (0, 1)$ or $I = (1, \infty)$.

Moreover, let us list a few publications on this subject [2–5].

[1] Nikol'skii S.M., Approximation of functions of many variables and theorems of embedding, Nauka, Moscow, 1977.

[2] Baidel'dinov B. and Oinarov R., Properties of one weighted space, In: Proc. Conf. "Function spaces, theory of approximations and non-linear analysis", Moscow 1995, 10–11.

[3] Kalybay A. and Oinarov R., Some properties of spaces with multi-weighted derivatives, In: Progress in Analysis, Vol. I, World Sci. Publ., 2003, 1–13.

[4] Abdikalikova Z., Baiarystanov A. and Oinarov R., Compactness of embedding between spaces with multiweighted derivatives – the case $p \leq q$, Math. Ineq. Appl., Vol. 14, 4 (2011), 793–810.

[5] Abdikalikova Z., Oinarov R. and L.-E. Persson, Boundedness and compactness of the embedding between spaces with multiweighted derivatives when $1 \leq q < p < \infty$, Czechoslovak Math. J., Vol. 61, 136 (2011), 7–26.

Examples of generalized gyrovector spaces

Osamu Hatori

Department of Mathematics, Fuculty of Sciences, Niigata University,
Niigata 950-2181, Japan
Email **hatori @math.sc.niigata-u.ac.jp**

We give examples of the generalized gyrovector space which is defined as follows.

Definition 1 (A generalized gyrovector space)**.** *Let* $(G, \oplus)$ *be a gyrocommutative gyrogroup with the map* $\otimes : \mathbb{R} \times G \to G$. *Let* ϕ *be an injection from* G *into a real normed space* $(\mathbb{V}, \|\cdot\|)$. *We say that* $(G, \oplus, \otimes, \phi)$ *(or* $(G, \oplus, \otimes)$ *just for a simple notation) is a generalized gyrovector space or a GGV in short if the following conditions (GGV0) to (GGV8) are fulfilled:*

(GGV0) $\|\phi(\mathrm{gyr}[\boldsymbol{u}, \boldsymbol{v}]\boldsymbol{a})\| = \|\phi(\boldsymbol{a})\|$ *for any* $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{a} \in G$;

(GGV1) $1 \otimes \boldsymbol{a} = \boldsymbol{a}$ *for every* $\boldsymbol{a} \in G$;

(GGV2) $(r_1 + r_2) \otimes \boldsymbol{a} = (r_1 \otimes \boldsymbol{a}) \oplus (r_2 \otimes \boldsymbol{a})$ *for any* $\boldsymbol{a} \in G,\ r_1, r_2 \in \mathbb{R}$;

(GGV3) $(r_1 r_2) \otimes \boldsymbol{a} = r_1 \otimes (r_2 \otimes \boldsymbol{a})$ *for any* $\boldsymbol{a} \in G,\ r_1, r_2 \in \mathbb{R}$;

(GGV4) $(\phi(|r| \otimes \boldsymbol{a}))/\|\phi(r \otimes \boldsymbol{a})\| = \phi(\boldsymbol{a})/\|\phi(\boldsymbol{a})\|$ *for any* $\boldsymbol{a} \in G \setminus \{\boldsymbol{e}\},\ r \in \mathbb{R}\setminus\{0\}$, *where* $\boldsymbol{e}$ *denotes the identity element of the gyrogroup* $(G, \oplus)$;

(GGV5) $\mathrm{gyr}[\boldsymbol{u}, \boldsymbol{v}](r \otimes \boldsymbol{a}) = r \otimes \mathrm{gyr}[\boldsymbol{u}, \boldsymbol{v}]\boldsymbol{a}$ *for any* $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{a} \in G,\ r \in \mathbb{R}$;

(GGV6) $\mathrm{gyr}[r_1 \otimes \boldsymbol{v}, r_2 \otimes \boldsymbol{v}] = id_G$ *for any* $\boldsymbol{v} \in G,\ r_1, r_2 \in \mathbb{R}$;

(GGVV) $\|\phi(G)\| = \{\pm\|\phi(\boldsymbol{a})\| \in \mathbb{R} : \boldsymbol{a} \in G\}$ *is a real one-dimensional vector space with vector addition* $\oplus'$ *and scalar multiplication* $\otimes'$;

(GGV7) $\|\phi(r \otimes \boldsymbol{a})\| = |r| \otimes' \|\phi(\boldsymbol{a})\|$ *for any* $\boldsymbol{a} \in G$, $r \in \mathbb{R}$;

(GGV8) $\|\phi(\boldsymbol{a} \oplus \boldsymbol{b})\| \leq \|\phi(\boldsymbol{a})\| \oplus' \|\phi(\boldsymbol{b})\|$ *for any* $\boldsymbol{a}, \boldsymbol{b} \in G$.

[1] T. Abe and O. Hatori, *Generalized gyrovector spaces and a Mazur-Ulam theorem*, to appear in Publ. Math. Debrecen

[2] A. A. Ungar, *Analytic Hyperbolic Geometry and Albert Einstein's Special Theory of Relativity*, World Scientific, (2008)

Sep–5 1520–1540

Coupled fixed point theorems for $\alpha - \psi -$Geraghty's contraction maps using monotone property

Narawadee Na Nan

Department of Mathematics, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand
Email **narawadee_n @hotmail.co.th**

We generalized the results presented by Kadelburg et at. (Fixed Point Theory Appl. 2015:27, 2015) and give other conditions to prove the existence and uniqueness of a fixed point of $\alpha - \psi -$Geraghty's contraction maps in complete metric spaces. An example illustrating our results is provided.

[1] M. Geraghty, *On contractive mappings*, Proc. Amer. Math. Soc. **40** (1973), 604–608.

[2] T. Gnana Bhaskar, V. Lakshmikantham, *Fixed point theorems in partially ordered metric spaces and applications*, Nonlinear Anal. TMA **65** (2006), 1379–1393.

[3] D. Guo, V. Lakshmikantham, *Coupled fixed points of nonlinear operators with applications*, Nonlinear Anal. TMA **11** (1987), 623–632.

[4] J. Harjani, B. López, K. Sadarangani, *Fixed point theorems for mixed monotone operators and applications to integral equations*, Nonlinear Anal. TMA **74** (2011), 1749–1760.

[5] E. Karapinar, *$\alpha - \psi -$Geraghty contraction type mappings and some related fixed point results*, Filomat 28:1 (2014), 37-48.

[6] E. Karapinar, *A discussion on "$\alpha - \psi -$Geraghty contraction type mappings"*, Filomat 28:4 (2014), 761-766.

[7] Z. Kadelburg, P. Kumam. S. Radenovic, W. Sintunavarat, *Common coupled fixed point theorems for Geraghty-type contraction mappings using monotone property*, Fixed Point Theory Appl. (2015) 2015:27.

[8] V. Lakshmikantham, Lj. Ćirić, *Coupled fixed point theorems for nonlinear contarctions in partially ordered metric spaces*, Nonlinear Anal. TMA **70** (2009), 4341–4349.

Subdifferentials of Nonconvex Integral Functionals in Banach Spaces: A Gelfand Integral Representation

Boris S. Mordukhovich and Nobusumi Sagara*

Department of Mathematics, Wayne State University
Detroit, MI 48202, USA
Email **boris @ math.wayne.edu**

Department of Economics, Hosei University
4342, Aihara, Machida, Tokyo, 194-0298, Japan
Email **nsagara @ hosei.ac.jp**

We investigate subdifferential calculus for nonconvex integral functionals on Banach spaces. To this end, we present a new approach in which the Clarke and Mordukhovich subdifferentials of an integral functional are represented as the Gelfand integral of the subdifferential mapping of a locally Lipschitzian integrand. We also introduce a stronger notion of nonatomicity, saturation, to invoke the Lyapunov convexity theorem for the Gelfand integral of the Mordukhovich subdifferential mapping. The main results are applied to stochastic DP with discrete time, where the differentiability of the nonconvex value function and the KKT condition with equality-inequality constraints are derived.

[1] Cascales, B., V. Kadets and J. Rodríguez, (2011). "The Gelfand integral for multi-valued functions", *J. Convex Anal.* **18**, 873–895.

[2] Clarke, F. H., (1983). *Optimization and Nonsmooth Analysis*, John Wiley & Sons, New York.

[3] Khan, M. A. and N. Sagara, (2015). "Maharam-types and Lyapunov's theorem for vector measures on locally convex spaces with control measures", forthcoming in *J. Convex Anal.* **22**.

[4] Mordukhovich, B. S., (2006). *Variational Analysis and Generalized Differentiation I: Basic Theory; II: Applications*, Springer, Berlin.

[5] Podczeck, K., (2008). "On the convexity and compactness of the integral of a Banach space valued correspondence", *J. Math. Econom.* **44**, 836–852.

[6] Sun, Y. N. and N. C. Yannelis, (2008). "Saturation and the integration of Banach valued correspondences", *J. Math. Econom.* **44**, 861–865.

Sep–5 1600–1620

Study on set-valued inequality based on set-valued analysis and covenx analysis

Shogo Kobayashi, Yutaka Saito, Tamaki Tanaka* and Syuuji Yamada

Department of Mathematics, Niigata University,
Niigata 950–2181, Japan

Email **tamaki @ math.sc.niigata-u.ac.jp**

Fan-Takahashi minimax inequality in [1, 4] is one of the important results in convex analysis as well as nonlinear analysis. In 2010, Kuwano, Tanaka and Yamada extend classical Fan-Takahashi minimax inequality into set-valued versions by using the following scalarizing functions for sets based on set-relations $\leq_C^{(j)}$ $(j = 1, \dots, 6)$ where C is a convex cone in a vector space:

$$(\mathrm{I}_{k,V}^{(j)} \circ F)(x) := \inf\left\{ t \in \mathbb{R} \;\middle|\; F(x) \leq_C^{(j)} (tk + V) \right\} \tag{1}$$

$$(\mathrm{S}_{k,V}^{(j)} \circ F)(x) := \sup\left\{ t \in \mathbb{R} \;\middle|\; (tk + V) \leq_C^{(j)} F(x) \right\} \tag{2}$$

where F is a set-valued map, $V \in 2^Y \setminus \{\emptyset\}$, direction $k \in \operatorname{int} C$ and the set-relations $\leq_C^{(j)}$ $(j = 1, \dots, 6)$.

In 2012, Saito, Tanaka and Yamada ([3]) propose a certain Ricceri's theorem ([2]) on Fan-Takahashi minimax inequality for set-valued maps with respect to "$\leq_C^{(5)}$."

These scalarizing functions have some kinds of montonicity and convexity, and certain inherited properties from a parent set-valued map: if set-valued map F has some kind of convexity, then $\mathrm{I}_{k,V}^{(j)} \circ F$ and $\mathrm{S}_{k,V}^{(j)} \circ F$ have also similar properies. In this talk, we take an overview of this kind of scalarization technique and show some applications to set-valued inequality.

[1] K. Fan, *A minimax inequality and its applications*, Inequalities III, O. Shisha (ed.), Academic Press, New York, 1972, pp.103–113.

[2] B. Ricceri, *Existence Theorems for Nonlinear Problems*, Rend. Accad. Naz. Sci. XL, **11** (1987), 77–99.

[3] Y. Saito, T. Tanaka, and S. Yamada, *On generalization of Ricceri's theorem for Fan-Takahashi minimax inequality into set-valued maps via scalarization*, J. Nonlinear and Convex Anal. **16** (2015), 9–19.

[4] W. Takahashi, *Nonlinear variational inequalities and fixed point theorems*, J. Math. Soc. Japan **28** (1976), 168–181.

Asymptotically quasi-nonexpansive mappings with respect to the Bregman distance

Yukino Tomizawa

Department of Mathematics, Faculty of Science and Engineering,
Chuo University, Tokyo 112-8551, Japan
Email **tomizawa @gug.math.chuo-u.ac.jp**

Many authors have studied self-mappings of nonexpansive type with respect to the Bregman distance in Banach spaces. However, few studies have attempted to study the case of nonself-mappings. The purpose of this study is to obtain an iterative approximation of fixed points of nonself-mappings of nonexpansive type with respect to the Bregman distance. We introduce the class of asymptotically quasi-nonexpansive nonself-mappings with respect to the Bregman distance. We construct an iteration scheme for approximating a fixed point of any mapping belonging to this class and prove a strong convergence theorem. This theorem is generalization of results of some authors [2, 3].

In this paper, $\mathbf{N}$ denotes the set of positive integers. We assume that E is a real reflexive Banach space. For a mapping $T : E \to E$, $F(T)$ denotes the set of fixed points of T. Let $f : E \to (-\infty, +\infty]$ be a convex function on E which is Gâteaux differentiable on the interior $\operatorname{int}\operatorname{dom} f$ of the effective domain $\operatorname{dom} f$ of f. A bifunction $D_f : \operatorname{dom} f \times \operatorname{int}\operatorname{dom} f \to [0, +\infty)$ given by $D_f(y, x) := f(y) - f(x) - \langle \nabla f(x), y - x\rangle$ is called the *Bregman distance with respect to* f (*cf.* [1]).

Definition 1. *Let C be a nonempty, closed, and convex subset of* $\operatorname{int}\operatorname{dom} f$*. Let T be a nonself mapping from C into* $\operatorname{int}\operatorname{dom} f$*. Let P be a nonexpansive retraction from* $\operatorname{int}\operatorname{dom} f$ *into C. The mapping T is said to be left Bregman asymptotically quasi-nonexpansive if $F(T) \neq \emptyset$ and there exists a sequence $\{k_n\} \subset [1, \infty)$, $k_n \to 1$ as $n \to \infty$ such that*

$$D_f(p, T(PT)^{n-1}x) \le k_n D_f(p, x) \quad \text{for } x \in C,\ p \in F(T),\ n \in \mathbf{N}.$$

[1] Bregman L. M. : *The relaxation method of finding the common point of convex sets and its application to the solution of problems in convex programming*, *USSR Comput. Math. Math. Phys.* **7** (1967), no. 3, 200–217.

[2] Wang S. and Kang S. M. : *Strong convergence iterative algorithms for equilibrium problems and fixed point problems in Banach spaces*, *Abst. Appl. Anal.* (2013), Art. ID 619762, 9 pp.

[3] Zhao L. C. and Chang S. S. : *Strong convergence theorems for a countable family of total quasi–asymptotically nonexpansive nonself mappings*, *Journal of Applied Mathematics* (2012), Art. ID 136134, 12 pp.

Sep–5 1700–1720

Shrinking projection methods with error for zero point problems in a Hilbet space

Takanori Ibaraki

Department of Mathematics Education Yokohama National University,
Tokiwadai, Hodogaya-ku, Yokohama 240-8501, Japan
Email **ibaraki @ynu.ac.jp**

In this talk, we study the shrinking projection method with error introduced by [1]. We obtain an iterative approximation of a zero point of a maximal monotone operator generated by the shrinking projection method with errors in a Hilbert space. Using our result, we discuss some applications.

[1] Y. Kimura, *Approximation of a common fixed point of a finite family of nonexpansive mappings with nonsummable errors in a Hilbert space*, J. Nonlinear Convex Anal. **15** (2014), 429–436.

[2] R. T. Rockafellar, *Monotone operators and proximal point algorithm*, SIAM J. Control. Optim. **14** (1976), 877–898.

[3] W. Takahashi, Y Takeuchi and R. Kubota, *Strong convergence theorems by hybrid methods for families of nonexpansive mappings in Hilbert spaces*, J. Math. Anal. Appl. **341** (2008), 276–286.

Sep–5 1720–1740

Upper bound of the number of all solutions for integer polynomial equations (modulo p^m)

Yi-Chou Chen

Department of General Education, National Army Academy, Taoyuan 320, Taiwan
Email **cycu.chou @gmail.com**

Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be an integer-valued polynomial function of the form

$$f(x) = \sum_{i=0}^{s} a_i x^i, \qquad \text{with degree of } x \text{ in } f(x) = s \geq 1,\ x \in \mathbb{Z}.$$

A prime p and a nonnegative integer m are given to find an integer $x \in \mathbb{Z}_{p^m}$ satisfying the following expression:

$$f(x) = 0\ (\textbf{mod}\ p^m), \qquad (*)$$

named as an integer polynomial equation modulo p^m.

In this paper, we will provide a non-NP hardness algorithm to solve all solutions and obtain a tide upper bound of the number of all solutions for this equation $(*)$.

Structure of Cesàro function spaces and interpolation

Lech Maligranda

Department of Engineering Sciences and Mathematics,
Luleå University of Technology, SE-971 87 Luleå, Sweden
Email **lech.maligranda @ ltu.se**

The *Cesàro function spaces* $Ces_p(I)$ on both $I = [0,1]$ and $I = [0,\infty)$ are classes of Lebesgue measurable real functions f on I such that the norm

$$\|f\|_{C(p)} = \left[\int_I \left(\frac{1}{x}\int_0^x |f(t)|dt\right)^p dx\right]^{1/p} < \infty, \quad \text{for } 1 \le p < \infty,$$

and

$$\|f\|_{C(\infty)} = \sup_{x\in I,\ x>0} \frac{1}{x}\int_0^x |f(t)|\,dt < \infty, \quad \text{for } p = \infty.$$

In the case $1 < p < \infty$ spaces $Ces_p(I)$ are separable, strictly convex and not symmetric. They, in the contrast to the sequence spaces, are not reflexive and do not have the fixed point property.

The structure of the Cesàro function spaces $Ces_p(I)$ was investigated in [1]–[3] and [6]–[7]. Their dual spaces, which equivalent norms have different description on $[0,1]$ and $[0,\infty)$, are described. The spaces $Ces_p(I)$ for $1 < p < \infty$ are not isomorphic to any $L^q(I)$ space with $1 \le q \le \infty$. They have "near zero" complemented subspaces isomorphic to l^p and "in the middle" contain an asymptotically isometric copy of l^1 and also a copy of $L^1[0,1]$. They do not have Dunford-Pettis property. Cesàro function spaces on $[0,1]$ and $[0,\infty)$ are isomorphic for $1 < p < \infty$. Moreover, the Rademacher functions span in $Ces_p[0,1]$ for $1 \le p < \infty$ a space which is isomorphic to l^2. This subspace is uncomplemented in $Ces_p[0,1]$. The span in the space $Ces_\infty[0,1]$ gives another sequence space.

In [5] and [8] it was shown that $Ces_p(I)$ is an interpolation space between $Ces_{p_0}(I)$ and $Ces_{p_1}(I)$ for $1 < p_0 < p_1 \le \infty$, where $1/p = (1-\theta)/p_0 + \theta/p_1$ with $0 < \theta < 1$. The same result is true for Cesàro sequence spaces. On the other hand, $Ces_p[0,1]$ is not an interpolation space between $Ces_1[0,1]$ and $Ces_\infty[0,1]$.

More general spaces were considered in [11]–[13]. For a Banach ideal function space X on I we define the *abstract Cesàro space* $CX = CX(I)$, the *abstract Copson space* $C^*X = C^*X(I)$ and the *abstract Tandori space* $\widetilde{X} = \widetilde{X}(I)$ as

$$CX = \{f \in L^0(I) : C|f| \in X\} \quad \text{with the norm } \ \|f\|_{CX} = \|\,C|f|\,\|_X,$$

$$C^*X = \{f \in L^0(I) : C^*|f| \in X\} \quad \text{with the norm} \quad \|f\|_{C^*X} = \|\, C^*|f|\, \|_X,$$
$$\widetilde{X} = \{f \in L^0(I) : \widetilde{f} \in X\} \quad \text{with the norm} \quad \|f\|_{\widetilde{X}} = \|\, \widetilde{f}\, \|_X,$$

where $Cf(x) = \frac{1}{x}\int_0^x f(t)\,dt$, $C^*f(x) = \int_{I\cap[x,\infty)} \frac{f(t)}{t}\,dt$ and $\widetilde{f}(x) = \operatorname{ess\,sup}_{t\in I,\, t\geq x}|f(t)|$.

Comparisons of Cesàro, Copson and Tandori spaces as well as the "iterated" spaces CCX and C^*C^*X are presented in [13]. It may happen that spaces are different but the corresponding Cesàro, Copson and Tandori spaces are the same, that is, there are $X \neq Y$ such that $CX = CY$, $C^*X = C^*Y$ and $\widetilde{X} = \widetilde{Y}$.

The duality of abstract Cesàro spaces was proved in [11]: under some mild assumptions on X we have $(CX)' = \widetilde{X'}$ in the case $I = [0,\infty)$ and $(CX)' = \widetilde{X'(w)}$, where $w(x) = \frac{1}{1-x}$, $x \in [0,1)$ in the case $I = [0,1]$

The real and complex interpolation methods of abstract Cesàro, Copson and Tandori spaces, including the description of Calderón-Lozanovskiĭ construction for those spaces were given in [12].

The investigations show an interesting phenomenon that there is a big difference between properties and interpolation of Cesàro function spaces in the cases of finite and infinite interval.

The talk is based on joint papers with Sergey V. Astashkin (Samara) and Karol Leśnik (Poznań).

[1] S. V. Astashkin and L. Maligranda, *Cesàro function spaces fail the fixed point property*, Proc. Amer. Math. Soc. 136 (2008), no. 12, 4289–4294.

[2] S. V. Astashkin and L. Maligranda, *Structure of Cesàro function spaces*, Indag. Math. 20 (2009), no. 3, 329–379.

[3] S. V. Astashkin and L. Maligranda, *Rademacher functions in Cesàro type spaces*, Studia Math. 198 (2010), no. 3, 235–247. "Open access" at: `http://journals.impan.pl/cgi-bin/doi?sm198-3-3`

[4] S. V. Astashkin, *On geometrical properties of Cesàro spaces*, Mat. Sb. 203 (2012), no. 4, 61–80 (Russian); English transl.: Sb. Math. 203 (2012), 514–533.

[5] S. V. Astashkin and L. Maligranda, *Interpolation of Cesàro sequence and function spaces*, Studia Math. 215 (2013), no. 1, 39–69.

[6] S. V. Astashkin and L. Maligranda, *A short proof of some recent results related to Cesàro function spaces*, Indag. Math. 24 (2013), 589–592.

[7] S.V. Astashkin and L. Maligranda, *Structure of Cesàro function spaces: a survey*, in: Proc. Conference "Function Spaces X", Poznań, Banach Center Publ. 102 (2014), 13–40. "Open access" at: `http://journals.impan.pl/cgi-bin/doi?bc102-0-1`

[8] S.V. Astashkin and L. Maligranda, *Interpolation of Cesàro and Copson spaces*, in: "Banach and Function Spaces IV", Proc. of the Fourth Internat. Symp. on Banach and Function Spaces (ISBFS2012) (12-15 Sept. 2009, Kitakyushu-Japan), Edited by M. Kato, L. Maligranda and T. Suzuki, Yokohama Publ., Yokohama 2014, 123–133.

[9] S. V. Astashkin and L. Maligranda, *Structure of Rademacher subspaces in the Cesàro type spaces*, Studia Math. 226 (2015), no. 3, 259–279.

[10] A. Kufner, L. Maligranda and L.-E. Persson, *The Hardy Inequality. About its History and Some Related Results*, Vydavatelski Servis Publishing House, Pilzen 2007.

[11] K. Leśnik and L. Maligranda, *Abstract Cesàro spaces. Duality.* J. Math. Anal. Appl. 424 (2015), no. 2, 932–951.

[12] K. Leśnik and L. Maligranda, *Abstract Cesàro spaces. Optimal range*, Integral Equations Operator Theory, 81 (2015), no. 2, 227-235. "Open access" at: `http://link.springer.com/article/10.1007%2Fs00020-014-2203-4`

[13] K. Leśnik and L. Maligranda, *Interpolation of abstract Cesàro, Copson and Tandori spaces*, submitted; Preprint of 22 pages submitted on 19 February 2015 at `arXiv:1502.05732v1`.

Pointwise multipliers on several function spaces

Eiichi Nakai

Department of Mathematics, Ibaraki University,
Mito, Ibaraki 310-8512, Japan
Email **enakai @ mx.ibaraki.ac.jp**

We denote by $L^0(\mathbb{R}^n)$ be the set of all measurable functions from $\mathbb{R}^n$ to $\mathbb{C}$. Let $E_1, E_2 \subset L^0(\mathbb{R}^n)$ be normed function spaces. We say that a function $g \in L^0(\mathbb{R}^n)$ is a pointwise multiplier from E_1 to E_2, if the pointwise multiplication fg is in E_2 for any $f \in E_1$. We denote by $\mathrm{PWM}(E_1, E_2)$ the set of all pointwise multipliers from E_1 to E_2. We abbreviate $\mathrm{PWM}(E, E)$ to $\mathrm{PWM}(E)$. In this talk we review several properties of pointwise multipliers. For example, we have the following.

Theorem 1. *If E is a Banach space and has the following property;*

$$f_n \to f \text{ in } E \ (n \to \infty) \quad \Longrightarrow \quad \exists \{f_{n(j)}\} \text{ (subsequence) s.t. } f_{n(j)} \to f \text{ a.e. } (j \to \infty), \tag{1}$$

then each $g \in \mathrm{PWM}(E)$ is a bounded operator.

Actually, from (1) we see that each pointwise multiplier is a closed operator. Hence it is a bounded operator by the closed graph theorem.

We review the results on pointwise multipliers on several function spaces, Lebesgue, Orlicz, Lorentz, Morrey, BMO, Campanato spaces, etc.

Sobolev's inequality for Riesz potentials of functions in central Herz-Morrey-Orlicz spaces on the unit ball

Yoshihiro Mizuta

Department of Mechanical Systems Engineering
Hiroshima Institute of Technology
2-1-1 Miyake,Saeki-ku,Hiroshima 731-5193, Japan
Email **y.mizuta.5x @ it-hiroshima.ac.jp**

We introduce central Herz-Morrey-Orlicz spaces on the unit ball, and study Sobolev's inequality for Riesz potentials of functions in central Herz-Morrey-Orlicz spaces on the unit ball.

[1] Y. Mizuta and T. Ohno, Sobolev's inequality for Riesz potentials in central Lorentz-Morrey spaces of variable exponent, RIMS Kôkyûroku Bessatsu **B43** (2013), 101–120.

[2] Y. Mizuta and T. Ohno, Sobolev's theorem and duality for Herz-Morrey spaces of variable exponent, Ann. Acad. Sci. Fenn. Math. **39** (2014), 389–416.

[3] Y. Mizuta and T. Ohno, Herz-Morrey spaces of variable exponent, Riesz potential operator and duality, Complex Var. Elliptic Equ. **60** (2015), no. 2, 211–240.

Monotonicity properties of Orlicz spaces equipped with the p-Amemiya norm

Marek Wisła

Faculty of Mathematics and Computer Science, Adam Mickiewicz University in Poznań,
Umultowska 87, 61-614 Poznań, Poland
Email **marek.wisla @ amu.edu.pl**

In 1932, while introducing a subclass of Banach spaces, W. Orlicz defined a norm by the formula $\|x\|_\Phi^o = \sup\left\{\int_T |x(t)y(t)|d\mu : y \in L_\Psi, I_\Psi(y) \le 1\right\}$, where Φ, Ψ are two Young functions conjugate to each other and $I_\Phi(x) = \int_T \Phi(x(t))d\mu$. In 1955 W.A.J. Luxemburg investigated the conjugate norm to the Orlicz one defined by $\|x\|_\Phi = \inf\left\{\lambda > 0 : I_\Phi\left(\frac{x}{\lambda}\right) \le 1\right\}$. H. Hudzik and L. Maligranda pointed out to the fact that Orlicz and Luxemburg norms are the bounder values of the family of (equivalent to each other) p-Amemiya norms defined by $\|x\|_{\Phi,p} = \inf_{k>0} \frac{1}{k} s_p(I_\Phi(kx))$, where $s_p : \mathbb{R}_+ \to \mathbb{R}_+$, $s_p(u) = (1 + u^p)^{1/p}$, for $1 \le p < \infty$ and $s_\infty(u) = \max\{1, u\}$ for $p = \infty$. During the talk strict monotonicity, lower and upper uniform monotonicities and uniform monotonicity of Orlicz spaces equipped with the p-Amemiya will be presented. It is worth noting that monotonicity properties can be directly applied to the best approximation problem.

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